EFFECTS OF TAIL GEOMETRIES ON THE PERFORMANCE AND WAKE PATTERN IN FLAPPING PROPULSION

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ABSTRACT
Swimming fishes exhibit remarkable diversities of the caudal fin geometries. In this work, a computational study is conducted to investigate the effects of the caudal fin shape on the hydrodynamic performance and wake patterns in flapping propulsion. We construct the propulsor models in different shapes by digitizing the real caudal fins of fish across a wide range of species spanning homocercal tails with low aspect ratio (square shape used by bluegill sunfish, rainbow trout, etc.) or high aspect ratio (lunate shape adopted by tuna, swordfish, etc.), and even heterocercal caudal fin adopted by sharks. Those fin models perform the same flapping motion in a uniform flow to mimic fish’s forward swimming. We then simulate the flow around the flapping fins by an in-house immersed-boundary-method based flow solver. According to the analysis of the hydrodynamic performance, we have found that the lunate shape model (high aspect-ratio) always generates a larger thrust compared to other models. The comparison of the propulsive efficiency shows that the large aspect ratio fins (tuna and shark) have a higher efficiency when the Strouhal number ($St$) is in the range of steady swimming (0.2<$St$<0.4), while the lower aspect ratio caudal fins (catfish, trout, etc.) are more efficient when $St$>0.4, in which the fish is accelerating or maneuvering. Finally, the 3D wake patterns of those propulsors are analyzed in detail.

NOMENCLATURE
\begin{itemize}
  \item $\bar{c}$: mean chord length
  \item $C_T$: thrust coefficient
  \item $C_P$: power coefficient
  \item $f$: flapping frequency
  \item $L$: span length
  \item $Re$: Reynolds number
  \item $S$: fin area
  \item $St$: Strouhal number
  \item $T$: flapping period
  \item $y_h$: lateral displacement of heaving motion
  \item $y_w$: lateral displacement of wavy motion
  \item $\varphi$: phase difference between the heaving and wavy motion
  \item $\lambda_w$: wave length
\end{itemize}

INTRODUCTION
Fishes in nature have a substantial diversity of the geometry of their caudal fins. It is thought that different geometric configurations may serve for different hydrodynamic purposes. For example, large aspect-ratio lunate caudal fins are usually adopted by fast swimmers which requires a high propulsive efficiency for their long-distance cruising. Low aspect ratio caudal fins are usually used by smaller fishes which desire a high maneuverability to escape from predators.

Biologists have contributed a large amount of literatures in quantifying the characteristics of the fin shape and their flapping kinematics [1-4]. People also studied the hydrodynamics of the flapping propulsors with different shapes separately [5-10].
There were little literatures systematically studied the effects of shape in flapping propulsion. Several basic parameters were used to characterize the shape, such as the aspect ratio ($AR$) and area moments. Dong et al. [11] studied the wake topology and hydrodynamic performance of pitching-plunging plate with different aspect ratio. Raspa et al. [12] examined the role of aspect ratio in self-propelled undulatory plates. Yeh and Alexeev [13] studied the effect of the aspect-ratio in plunging flexible plates. All those studies found that the varying of aspect ratio will significantly change the propulsive performance and wake pattern. Li and Lu [14] numerically investigated the dynamic performance and the wake patterns of canonical plates with different shapes and found that the thrust behavior is related to the area moment of the plate.

In the flapping propulsion, the change of the shape not only alters the geometric characteristics, but also changes the kinematic features. For example, the flapping amplitude along the trailing edge of the propulsor will vary as the trailing edge shape changes even though the peak-to-peak amplitude of the tip is kept the same. So in the selection of the basic control parameters, we should consider the change of the dynamic features with the geometry. In this paper, considering the most important dynamic control parameters in flapping propulsion are the Reynolds number and the Strouhal number, we first coordinate the shape effect with the definition of these two parameters. We select seven typical caudal fins, spanning homocercal tails with low aspect ratio (square shape used by bluegill sunfish, rainbow trout, etc.) or large aspect ratio (lunate shape adopted by tuna, swordfish, etc.), and even heterocercal caudal fin adopted by sharks. The fins are modeled as thin plates with the thickness of only 3% of their chord length. To simplify the problem and make a fair comparison of the different shapes, the model fins are performing the same flapping kinematics, which is digitized from the steady swimming of a typical subcarangiform swimmer (bluegill sunfish). The fin models are put in a uniform flow to mimic the fish’s forward swimming. The three-dimensional incompressible Navier-Stokes equations are solved with an immersed-boundary-method based flow solver. The hydrodynamic performance (thrust, power and propulsive efficiency) and the wake patterns are carefully analyzed and compared.

### METHODOLOGY

#### Physical model

We digitize the caudal fin (or fluke) profiles of several common fishes (or mammal) as shown in Figure 1. Those models are cross a wide range of fish/mammal species, from fresh water fish to marine animals; from low aspect-ratio ($AR$) to high aspect ratio tails; from square shape to forked shape and eventually to lunate geometry. Aspect-ratio is one of the important parameters in quantifying the propulsor shape. It is defined as follows [11],

$$AR = \frac{L^2}{S}$$

where $S$ and $L$ are the area and the span length of the propulsor, respectively. We summarize $AR$ of those propulsor models in Table 1, in which the catfish has the lowest $AR$ while the tuna has the highest one.

![Figure 1. Propulsor models with different shapes digitized from the real fish or aquatic mammals.](image)

<table>
<thead>
<tr>
<th>species</th>
<th>catfish</th>
<th>trout</th>
<th>sunfish</th>
<th>bluefish</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AR$</td>
<td>1.218</td>
<td>1.951</td>
<td>2.137</td>
<td>2.802</td>
</tr>
<tr>
<td>species</td>
<td>dolphin</td>
<td>shark</td>
<td>tuna</td>
<td></td>
</tr>
<tr>
<td>$AR$</td>
<td>2.815</td>
<td>3.503</td>
<td>6.652</td>
<td></td>
</tr>
</tbody>
</table>

To make a fair comparison among those models, we employ the same flapping kinematics to them. Based on the high-speed videos of a sunfish in steady swimming [15], the flapping motion can be decomposed into two parts, i.e., a heaving motion and wavy motion, which are expressed as follows,

$$y = y_h + y_w$$

where $y$ represents the lateral displacement; $y_h$ and $y_w$ are the displacement due to the heaving and wavy motion, respectively.

$$y_h = A_h \cdot \sin \left(\frac{2\pi t}{T} + \varphi\right)$$

$$y_w = A_w \cdot x \cdot \sin \left(\frac{2\pi}{\lambda_w} \left(\frac{x}{\lambda_w} - \frac{t}{T}\right)\right)$$

In equation (3) and (4), $A_h$ is the heaving amplitude; $A_w$ the coefficient of the waving amplitude; $\lambda_w$ wavelength; $\varphi$ the phase difference between heaving and the waving; $t$ is time; $T$ is the flapping period. The values of those parameter employed in this paper are shown in Table 2.
The numerical methodology of the immersed-boundary-method-based Navier-Stokes equation solver employed in the current study is briefly introduced here. The 3D incompressible Navier-Stokes equations are discretized using a cell-centered, collocated arrangement of the primitive variables, and is solved using a finite difference-based Cartesian grid immersed boundary method [18]. The equations are integrated in time using the fractional step method. A second-order central difference scheme is employed in space discretization. The Eulerian form of the Navier-Stokes equations is discretized on a Cartesian mesh and boundary conditions on the immersed boundary are imposed through a ghost-cell procedure. This method was successfully applied in many simulations of flapping propulsion [19-27]. More details about this method can be found in references [11, 18]. Validations about this solver can be found in our previous works [20, 28].

The simulations were carried out on a non-uniform Cartesian grid. The computational domain size was chosen as 15c×10c×10c with 400×160×192 (about 12.3 millions) grid points in total. High-resolution uniform grids around the fin models with the spacing of about 0.015c is designed to resolve the near-field vortex structures. At the left-hand boundary, we provide a constant inflow velocity boundary condition. The right-hand boundary is the outflow boundary, allowing the vortices to convect out of this boundary without significant reflections. The zero gradient boundary condition is provided at all lateral boundaries. A homogeneous Neumann boundary condition is used for the pressure at all boundaries.

RESULTS

In this section, we will first compare the results (hydrodynamic performance and the flow structures) of two typical fin models, i.e., trout and tuna. And then we show the results of all the models examined in this paper.

Trout vs. tuna

In this sub-section, we compare the hydrodynamic performance and wake patterns between trout and tuna caudal fin models at Str=0.35 and Re=2500. We simulate the cases for six flapping cycles so that both the hydrodynamic forces and the flow structure exhibit a good periodicities. The results shown in the following are in the last flapping cycle.

Table 3. Hydrodynamic performance of trout and tuna caudal fin models

<table>
<thead>
<tr>
<th></th>
<th>trout</th>
<th>tuna</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{C}_T$</td>
<td>0.071</td>
<td>0.211</td>
</tr>
<tr>
<td>$\tilde{C}_p$</td>
<td>0.435</td>
<td>0.836</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.164</td>
<td>0.252</td>
</tr>
</tbody>
</table>
Figure 3 shows the comparison of time variation of $C_T$ and $C_P$ between trout and tuna caudal fins and the cycle averaged values of them are shown in Table 1. From Figure 3, both the force and power coefficients exhibit half-cycle periodicity because the flapping motion (left stroke and right stroke) is symmetric in our model. There are two peaks and two troughs of thrust coefficient during one flapping cycle for both trout and tuna. The trough values in these two models are close to each other, however, the peak values of tuna is much higher than those of trout. This leads to a 190% increase of the mean thrust coefficient of tuna compared to that of trout (see Table 3). The mean power coefficient in tuna caudal fin model is nearly twice as much as that in trout. The propulsive efficiency of the tuna caudal fin is about 0.252, which is about 54% higher than that of trout.

The 3D vortex structures are visualized by the isosurface of the imaginary part of the complex eigenvalue ($\lambda$) derived from the instantaneous velocity gradient tensor, which identifies flow regions where rotation dominates over strain [29-31]. The flow structures of trout and tuna caudal fin models are shown in Figure 4 and Figure 5, respectively. The overall feature of the
The flow structures in these two models are similar to each other. Both figures show that, at each stroke (left stroke or right stroke), there is a vortex ring shed from the fin and the vortex rings are inter-connected with each other. The backward flow induced by these vortex rings is responsible for the thrust production. This is in line with previous studies on the three-dimensional vortex structures of fish tail [32, 33].

The difference of the 3D flow structures between trout and tuna is obvious. From the top view (Figure 4 (a) and Figure 5 (a)), we can see that the vortex loop is expanding as they propagate backward in both trout and tuna models, but the expanding angle in the tuna (12°) is less than that of trout (18°). The lower expanding angle may lead to a more concentrated backward flow, which explains the higher efficiency of the tuna model.

The other obvious difference between those two models is that, in trout model, the vortex ring seems compressed in the vertical direction right after the shedding, so that the vertical vortex tube shed from the trailing edge is tilted immediately. Though the vortex rings is somewhat compressed in tuna model, we can observe the vertical vortex tubes right behind the trailing edge. This feature makes the wake have a stronger concentrated vorticity, which is helpful to the thrust production. This is much clearer when we show the vertical vorticity on the mid-plane of these two cases in Figure 6. The 2D wake of the tuna caudal fin is similar to that of a 2D model fish’s propulsion [34-37]. It is known that the 2D model can generate more thrust than that of 3D model when the kinematics is the same.

**Comparison of seven caudal fin models**

In this sub-section, we simulate the hydrodynamics of the seven caudal fin models shown in Figure 1. The Reynolds number is fixed at Re=2500 while the Strouhal number is varying from 0.25 to 0.6.

![Figure 7](image)

**Figure 7.** (a) Thrust coefficient and (b) propulsive efficiency of seven caudal fin models.

Figure 7(a) shows the thrust coefficient of those seven caudal fin models. It demonstrates that the thrust is increasing as St increases in all models. According to this figure, the thrust is highly dependent on the aspect-ratio. The lowest AR model (catfish) has the lowest thrust and the highest AR model (tuna) generates the largest thrust at the same Strouhal number. The trend of the propulsive efficiency with respect to St is different with that of thrust coefficient. According to Figure 7(b), we found that the larger AR propulsors (tuna and shark) are more efficient when St < 0.4, while the lower AR propulsors (catfish, trout, etc) have higher propulsive efficiency when St > 0.4. We know that the Strouhal number of aquatic animals’ cruising is in the range of 0.2 ~ 0.4 [38]. The higher St usually corresponds to the accelerating motion or maneuvering, in which the swimming speed is slower than the steady swimming. Our results indicate that it is better to adopt a large AR propulsor when cruising at a...
high speed, while a low AR fin is more efficient when accelerating or maneuvering. This somewhat explains why the high speed cruising fishes in the ocean, such as tuna, shark, sailfish, etc., usually possess large AR caudal fins, but some smaller fresh water fish such as sunfish and trout, which need a high maneuverability to escape from predators, more likely employ low AR fins.

CONCLUSIONS

The hydrodynamic performance and wake structures of flapping propulsors with different shapes have been investigated by solving the 3D Navier–Stokes equations using an immersed boundary method. The propulsor models are digitized from the real fish’s caudal fins and the flapping kinematics of a subcarangiform swimmer (sunfish) is applied to all those models. Our simulation results demonstrate that the lunate shape model (large aspect-ratio) always generates a larger thrust compared to other models. The comparison of the propulsive efficiency indicates that the large AR fins (tuna and shark) has a higher efficiency during cruising ($St < 0.4$), however, low aspect ratio caudal fins (catfish, trout, etc.) is more efficient when fish are accelerating or maneuvering ($St > 0.4$). The 3D wake patterns of those propulsors are analyzed in detail. We found that vortex shapes are highly dependent on the fin aspect-ratio and the trailing edge shape. This work provides a physical insight into the understanding of hydrodynamics and flow structures for the propulsor with different geometries and provides guidelines to the design of propulsors for bioinspired autonomous underwater vehicles.

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