Virtual Force Measurement of POD Modes for a Flat Plate in Low Reynolds Number Flows

Zongxian Liang\(^1\) and Haibo Dong\(^2\)
Department of Mechanical & Aerospace Engineering,
University of Virginia, Charlottesville, VA 22904

A POD Mode Force Survey Method (POD-FSM) is presented to measure virtual forces of POD modes acting on a flat plate in low Reynolds number flows. In particular, a pitching-plunging plate is used to examine major aspects of the POD-FSM. These include the effect of the number of POD modes and dependence of computational domains. The results have shown that the superposition of virtual forces of full POD modes can accurately predict the aerodynamic forces acting onto the plate, whereas the virtual forces of six high-energy contained POD modes can also measure the thrust with only 5\% error. It is also found that the virtual forces of POD modes are sensitive to the size of the computational domain.

Nomenclature

\( F \) = aerodynamic force acting on bodies
\( F_i \) = force term of the \( i^{th} \) POD mode
\( F_{ij} \) = force term caused by the interaction of the \( i^{th} \) and \( j^{th} \) POD mode
\( F_{u,j} \) = force term caused by the interaction of the \( i^{th} \) POD mode and bodies
\( \tilde{r} \) = position vector
\( U \) = velocity snapshot and exact velocity field
\( \overline{U} \) = mean (time averaged) velocity over snapshots
\( u_i \) = fluctuating velocity field
\( \alpha_i \) = temporal coefficient of the \( i^{th} \) POD mode
\( \Phi_i \) = POD base element of velocity
\( \Psi_i \) = vorticity of POD modes, curl of \( \Phi_i \)

I. Introduction

It is attractive to researchers and engineers to effectively and efficiently conduct physics-based analysis in a variety of fluid dynamics studies, e.g. flapping wing aerodynamics. Among all methods, Direct Numerical Simulation (DNS) has been validated\(^{[1-6]}\) and demonstrated as an effective and accurate approach. However, the time cost of DNS is expensive for any computer-intensive applications, especially for three-dimensional problems. Thus, to accurately predict the dynamics of flow problems such as flow control at a low computational cost, Reduced-Order Models (ROMs)\(^{[7]}\), specifically Proper Orthogonal Decomposition (POD) based ROMs\(^{[8]}\), have been widely used.

In previous studies\(^{[9, 10]}\), a pressure corrected ROM method has been developed and proved valid for flapping flight. However, the aerodynamic performance of the ROMs was hardly discussed. This is mainly because there is a lack of tools relating the aerodynamic force of flapping plates with the POD

---

\(^1\) Ph.D. Student, AIAA student member, zl3ra@virginia.edu
\(^2\) Associate Professor, AIAA Associate Fellow, haibo.dong@virginia.edu

American Institute of Aeronautics and Astronautics Paper 2014-0054
modes of corresponding unsteady flows. In general, POD modes are considered as statistically representative of the average kinetic energy of flows\textsuperscript{[11]}. Hence, traditional analyses of POD modes are based on the sense of energy capture\textsuperscript{[12]}.

As required by a large number of ROM applications which focus on optimizing aerodynamic force of flows\textsuperscript{[13]}, a method describing the direct connection between the aerodynamic forces and POD modes of unsteady flows is necessary. To this end, we present a POD Mode Force Survey Method (POD-FSM) that can be used to measure virtual forces of POD modes of low Reynolds number flows obtained by DNS. The results have shown the superposition of these virtual forces of full POD modes can accurately reconstruct the aerodynamic forces of a pitching-plunging plate in Reynolds number 200 flows. An outline of the chapter is given below. In Section II, a brief introduction to the methodology of DNS and POD is given first, followed by the theorem of the impulse equation\textsuperscript{[14]} and the POD-FSM. In Section III, we first present a flow past a stationary plate to validate the impulse equation. Next, the POD-FSM is applied to flows past a plate undergoing pitching-plunging motions. Effect of the number of POD modes to the POD-FSM and computational domain dependence of the POD-FSM are studied. Virtual forces of each individual POD modes are also examined.

II. Methodology

A. Direct Numerical Simulation

The non-dimensional incompressible Navier-Stokes equations, as written in Eq. (1),

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i \partial x_j}
\]

are discretized on a Cartesian mesh using second-order central difference scheme in space. A second-order accurate fractional-step method for time advancement is employed. Its key feature is that simulations with complex boundaries can be carried out on stationary non-body conformal Cartesian grids, eliminating the need for complicated re-meshing algorithms that are usually employed with conventional Lagrangian body-conformal methods. The boundary conditions on the immersed body are imposed through a “ghost-cell” procedure\textsuperscript{[15]}. The pressure Poisson equation is solved using the geometric multi-grid method integrating with the immersed-boundary methodology.

B. Proper Orthogonal Decomposition (POD)

Proper Orthogonal Decomposition is a method used to represent large data fields with a relatively small number of elements. POD creates a set of basis function which spans the original data set by capturing the characteristic components. The system can be represented by the first few dominant modes.

Let \( \{ U_i(x): 1 \leq i \leq N, x \in \Omega \} \) be a set of \( N \) snapshots of a domain \( \Omega \). The mean (time averaged) velocity of the snapshots is given by \( \bar{U} = \frac{1}{N} \sum_{i=1}^{N} U_i(x) \). Thus the fluctuating velocity is \( u_i = U_i - \bar{U} \), \( i = 1, \ldots, N \). To form a set of the modes \( \Phi_i \) in the context of proper orthogonal decomposition, it is required to maximize the quantity

\[
\frac{\langle (u, \Phi)^2 \rangle}{\| \Phi \|^2}
\]

subjected to a constraint \( \| \Phi \| = 1 \) for each mode \( \Phi_i \) in the set. Here, \( (f, g) \) is an inner product defined as

American Institute of Aeronautics and Astronautics Paper 2014-0054
\[(f,g) = \int_\Omega f(x)g(x)dx, \quad \|f\| = (\int_\Omega f(x)^2 dx)^{1/2}, \quad \text{and } \langle f \rangle \text{ is a time average operation on the data ensemble.} \]

The POD method in this scenario produces the POD modes \( \Phi_i \) that maximize the turbulent kinetic energy. The optimization problem can be transformed into an eigenvalue problem by using the snapshot method, written as:

\[ AV = \lambda V \]  

(3)

where the entry of matrix \( A \) is \( a_{ij} = \langle u_i, u_j \rangle \).

After solving the eigenvalue problem, the eigenvalue \( \lambda_i \) and corresponding eigenvector \( V_i \) are usually reorganized in descending order. Each POD basis element \( \Phi_i \) is given by \( \Phi_i = \sum_{j=1}^{N} a_{ij}' u_j \) where \( a_{ij}' \) is the \( j^{th} \) element of the \( i^{th} \) eigenvector \( V_i \). Based on \( \Phi_i \), any snapshot can then be reconstructed using a linear combination of the POD basis elements \( U^n = \bar{U} + \sum_{i=1}^{N} \alpha^n_i \Phi_i \) where \( \alpha^n_i = \Phi_i \cdot u_n \).

C. The Impulse Equation and the POD-FSM

Noca et al.\[14\] presented an impulse equation to measure instantaneous forces on immersed bodies with only information of velocity and vorticity,

\[ \vec{F} = -\frac{1}{N-1} \frac{d}{dt} \int_{V(t)} \vec{r} \times \vec{\omega} dV + \int_{S(t)} \vec{n} \cdot \gamma_{imp} dS \]

(4)

\[ + \frac{1}{N-1} \frac{d}{dt} \int_{S_b(t)} \vec{r} \times (\vec{n} \times \vec{u}) dS - \int_{S_b(t)} \vec{n} \cdot (\vec{u} - \vec{u}_s) \vec{u} dS \]

\[ \gamma_{imp} = \frac{1}{2} u^2 I - \vec{u} \vec{u} - \frac{1}{N-1} (\vec{u} - \vec{u}_s) (\vec{r} \times \vec{\omega}) + \frac{1}{N-1} \vec{\omega} (\vec{r} \times \vec{u}) \]

\[ + \frac{1}{N-1} \left[ \vec{r} \cdot (\nabla \cdot T) I - \vec{r} (\nabla \cdot T) \right] + T \]

where \( \vec{n} \) is the normal vector of surfaces, \( I \) is the unit tensor, \( \vec{u}_s \) is velocity of the control surface and \( T \) is the viscous stress tensor \( T = \mu \left( \nabla \vec{u} + \nabla \vec{u}^\top \right) \). The control surface integral term \( \gamma_{imp} \) in (4) includes the information of external boundaries. The first two terms in \( \gamma_{imp} \) represent the Kutta-Zhukovsky or vortex force.

If we consider POD modes as a special type of vortex structure in terms of kinetic energy and assume that each POD mode is associated with a virtual force that can be measured by the impulse equation, we can have a force measurement for POD modes. This method, termed POD Mode Force Survey Method (POD-FSM), can be derived by substituting the POD expressions of velocity and vorticity into the impulse equation (4). Because the body surface is impermeable, the surface integral term \( \int_{S_b(t)} \vec{n} \cdot (\vec{u} - \vec{u}_s) \vec{u} dS \) can be omitted. As a result, one obtains the force expression in terms of

\[ F = \sum_{i=0}^{N} \left( F_i + F_{b,i} \right) + \sum_{j=0}^{N} \sum_{j=0}^{N} F_{ij} \]

(5)

where \( F_i \) are force of the \( i^{th} \) POD mode, \( F_{ij} \) are force caused by the interaction between the \( i^{th} \) and \( j^{th} \) POD mode and \( F_{b,j} \) are the force caused by the interaction between the \( i^{th} \) POD mode and the body.

\( F_i \) is a non-interaction term consisting of one volume integral term related to the first moment of POD vorticity and three surface integral terms related to viscous stress tensor \( T \), written as

\[ \text{American Institute of Aeronautics and Astronautics Paper 2014-0054} \]
\[ F_i = -\frac{1}{N-1} \frac{d\alpha_i}{dt} \int_{S(t)} \vec{r} \times \Psi_i dV + \alpha_i \int_{S(t)} \vec{n} \cdot \chi_i dS \]  \hspace{1cm} (6)

where \( \Psi_i \) is the curl of POD modes \( \Phi_i \), \( \Psi_i(\vec{r}) = \nabla \times \Phi_i(\vec{r}) \),

\[ \chi_i = \frac{1}{N-1} \left[ \vec{r} \cdot \left( \nabla \Phi_i(\vec{r}) \right) I - \vec{r} \left( \nabla \cdot \Phi_i(\vec{r}) \right) \right] + T_i \]  \hspace{1cm} (7)

\[ T_i = \mu \left[ \nabla \Psi_i(\vec{r}) + \nabla \Phi_i(\vec{r})^T \right] \]

Note that the mean velocity is denoted as mode zero \((i=0)\), whose temporal coefficient \( \alpha_0 \) is equal to one.

The interaction terms \( F_{ij} \) include products of different POD modes representing the original terms \( u^2 I \), \( \vec{u} \), \( (\vec{u} - \vec{u}_s)(\vec{r} \times \vec{\omega}) \) and \( \vec{\omega}(\vec{r} \times \vec{u}) \) in the impulse equation. Thus, \( F_{ij} \) are the force caused by the interaction between the \( i^{th} \) and \( j^{th} \) POD mode at the external surface of control volume.

\[ F_{ij} = \alpha_i \alpha_j \int_{S(t)} \vec{n} \cdot \eta_{ij} dS \]  \hspace{1cm} (8)

where

\[ \eta_{ij} = \frac{1}{2} \left[ \Phi_i \Phi_j \right] I - \Phi_i \Phi_j \]

\[ -\frac{1}{N-1} (\Phi_i - \vec{u}_s)(\vec{r} \times \Psi_j) \]  \hspace{1cm} (9)

\[ + \frac{1}{N-1} \Psi_i(\vec{r} \times \Phi_j) \]

Immersed bodies may move in fluids. This movement interacts with each POD mode and generates additional forces, written as

\[ F_{bi} = \frac{1}{N-1} \frac{d}{dt} \int_{S(t)} \vec{r} \times \left[ \vec{n}(t) \times \alpha_i \Phi_i \right] dS \]  \hspace{1cm} (10)

If the immersed body is a zero-thickness plate, \( F_{bi} \) is equal to zero because the flow velocity is the same at the upper and lower surfaces as the moving velocity of the plate. The normal vectors of the upper and lower surfaces are opposite. As a result, the surface integral around the plate is zero.

### III. Results

#### A. Validation of the Impulse Equation

The impulse equation\(^{14}\) is validated by a flow past a two-dimensional stationary membrane plate. A two-dimensional plate (zero thickness) is placed in flow fields with angle of attack equal to 30°. Incoming free stream of velocity \( U_\infty = 1 \). The chord length of the stationary membrane plate is one. The Reynolds number of the flow is 200. The size of flow domain is 18 (X) \times 15 (Y) with the grid number equal to 353 \times 258. The velocity boundaries are of Dirichlet boundary condition except at the right-hand boundary where it is outflow. Homogeneous Neumann boundary condition is specified on all the boundaries for pressure. The history of drag and lift coefficients, which begins from an impulse start and lasts for 40 units of non-dimensional time, is shown in Fig. 1. Table 1 lists time-average and root mean square (RMS) of the drag and lift coefficients. The DNS result is computed by surface integral of the pressure and viscous terms around the plate, while the computation of the impulse equation uses a domain size equal to the simulation domain. Smaller computational domains of the impulse equation may change the force magnitude up to 10%, which is in a range similar to the results given in\(^{17}\). In general, despite that the impulse method slightly underestimates the lift and overestimates the drag, it shows a good agreement with the DNS result in both lift and drag curves.
Figure 1. Force coefficients history from $t/T=0$ to 40. The legend ‘Impulse’ means the impulse equation.

Table 1: Comparison of drag and lift coefficients between DNS and the impulse equation

<table>
<thead>
<tr>
<th></th>
<th>DNS</th>
<th></th>
<th>Impulse</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>RMS</td>
<td>Average</td>
<td>RMS</td>
</tr>
<tr>
<td>$C_D$</td>
<td>0.74</td>
<td>0.74</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$C_L$</td>
<td>1.14</td>
<td>1.14</td>
<td>1.11</td>
<td>1.12</td>
</tr>
</tbody>
</table>

B. Flow past a membrane plate undergoing a pitching-plunging motion

Direct numerical simulations were conducted for flows past a two-dimensional, membrane plate undergoing the pitching and plunging motion. The kinematics of the plate can be prescribed by

$$y(t) = -H \cos(2\pi ft)$$

$$\alpha(t) = A \sin(2\pi ft)$$

where $H$ is the heave amplitude, $A$ is the amplitude of the sinusoidal pitch angle variation and $f$ is frequency. The pitch angle amplitude $A$ is equal to 30°. Strouhal number and Reynolds number are defined, respectively, as $St=2Hf/U_\infty$ and $Re=U_\infty c/\nu$, where $U_\infty$ is the uniform flow velocity, $c$ is the chord length of the plate and $\nu$ is the kinematics viscosity of fluids. In the current study, the heave amplitude $H$ is equal to 0.5, the uniform flow velocity $U_\infty$ is one, and the frequency is in a range of $0.4 \leq f \leq 0.6$. Thus $St$ ranges from 0.4 to 0.6. Without loss of generality, we specified a representative low Reynolds number equal to 200 for the study. The configuration of the flow domain is the same as the stationary case. The plate is placed at the middle of the flow field, as shown in Fig.1.

The number of POD modes and the size of computational domain are two factors that can affect the aerodynamic force calculated by the POD-FSM. Their effect on the accuracy of the POD-FSM are investigated in the current study. The flow reached a periodic shedding state in approximately 2-3 periods after startup. We took the 6th period as POD data ensemble to extract 96 snapshots every 10 frames (960 frames per period) and generate 2, 6, 12, 24, 48 and 96 POD modes. Two regions were used in the POD analysis and the POD-FSM. Region R1 is
a rectangular domain with the lower left corner being (-3, -4) and the upper right corner being (5, 4) (see Fig.1 for reference). Region R2 is defined by moving the upper right corner to (3, 4). In addition, to examine the usefulness of the POD-FSM to the pitching-plunging flows at a range of Strouhal numbers, we chose three representative frequency \( f = 0.4, 0.5 \) and 0.6. Table 2 lists a number of selected cases with different combinations of the number of POD modes, the regions and \( St \).

Fig. 2 shows a comparison of vorticity contour of the flow at \( St = 0.6 \) between the DNS result and the reconstruction using 96 POD modes. It can be seen that the major features of the reconstructed flow are similar to the DNS result, except that near the trailing edge of the flapping plate the shape of the vortex on the upper surface is slightly distorted.

<table>
<thead>
<tr>
<th>( St )</th>
<th># of POD Modes</th>
<th>Region</th>
<th>RMS of Thrust</th>
<th>Error of RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>96</td>
<td>R1: [-3,5] × [-4,4]</td>
<td>1.91</td>
<td>-4.0%</td>
</tr>
<tr>
<td></td>
<td>48</td>
<td></td>
<td>1.96</td>
<td>-1.5%</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td>1.96</td>
<td>-1.5%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td>1.94</td>
<td>-2.0%</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td>1.89</td>
<td>-5.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>1.64</td>
<td>-17.6%</td>
</tr>
<tr>
<td></td>
<td>96</td>
<td>R1: [-3,5] × [-4,4]</td>
<td>1.96</td>
<td>-1.5%</td>
</tr>
<tr>
<td></td>
<td>48</td>
<td></td>
<td>2.04</td>
<td>+2.5%</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td>2.04</td>
<td>+2.5%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td>2.04</td>
<td>+2.5%</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td>2.03</td>
<td>+2.0%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>1.65</td>
<td>-17.1%</td>
</tr>
<tr>
<td>0.5</td>
<td>96</td>
<td>R1: [-3,5] × [-4,4]</td>
<td>1.14</td>
<td>-5.8%</td>
</tr>
<tr>
<td>0.4</td>
<td>96</td>
<td>R1: [-3,5] × [-4,4]</td>
<td>0.61</td>
<td>-3.2%</td>
</tr>
</tbody>
</table>

Figure 2. Vorticity contours at \( t/T = 1 \) for the flow at \( St = 0.6 \).

Contribution of individual eigenvalue to the total turbulent kinetic energy can be expressed as normalized eigenvalues in a form of \( \frac{\lambda_i}{\sum_{k=1}^{N_x} \lambda_k} \). Captured kinetic energy by the first \( i \) modes can be represented by \( \frac{\sum_{k=1}^{i} \lambda_k}{\sum_{k=1}^{N_x} \lambda_k} \). The eigenvalue spectrum of the flows at \( St = 0.4, 0.5 \) and 0.6 is shown in Fig. 3. The decay of eigenvalues of three cases follows the same trend. First two modes contain nearly 88% of
total turbulent kinetic energy. The accumulation of first six modes increases up to 97%. Higher order
POD modes contain trivial fluctuating kinetic energy that mainly concentrates near the flapping plate.

The thrust and lift coefficients are defined as

\[ C_T = \frac{2F_T}{U_c^2c} \quad (13) \]

\[ C_L = \frac{2F_L}{U_c^2c} \quad (14) \]

where \( F_T \) and \( F_L \) are thrust and lift acting on the plate, respectively. The force history of the flows
at \( St=0.4, 0.5 \) and 0.6 is shown in Fig. 4. Comparing to the result in [2] which used an ellipsoidal foil with thickness ratio equal to 0.12,
the zero-thickness plate causes relatively larger peak forces in both lift and drag. As \( St \) decreases,
the peak force of thrust and lift decreases significantly. The time averaged thrust coefficient
of the flows at \( St=0.4, 0.5 \) and 0.6 is 0.44, 0.92 and 1.56, respectively; and their RMS is 0.63,
1.21 and 1.99, respectively. The impulse equation and the POD-FSM were applied to the flows and both methods show a good agreement with the
DNS result. When all 96 POD modes are used, the forces calculated by the POD-FSM converge to the corresponding results of the impulse equation in
all three cases. Comparing with the DNS results, the RMS error of thrust is no greater than 6%, as
indicated in Table 2.

Figure 3. Eigenvalue spectrum of the flows at \( St=0.4, 0.5 \) and 0.6.

Figs. 5a and 5b show, respectively, the thrust and lift coefficients of the DNS result and reconstructed
forces with different number of POD modes in Regions R1 for the case at \( St=0.6 \). The thrust reconstructed
with 12 or more modes agrees well with the DNS result. The curve of 6 modes has a slightly different
trend, comparing with the DNS result. The curve of 2 modes is trigonometric-like wave with the
amplitude and timing of peaks and valleys greatly deviating from the DNS. In contrast, the reconstructed
lift of 6 modes shows excellent agreement with the DNS result and the 2-mode case still follows the
general trend.

From the perspective of statistical measurement in one period, the RMS thrust of 96 modes is 1.91,
which is 4% smaller than the DNS result. Decreasing the number of POD modes from 96 to 12 slightly
reduces the error. The error of 6 modes increase to 5.0%, and the thrust of 2 modes is 18% smaller than

7    American Institute of Aeronautics and Astronautics Paper 2014-0054
the DNS. It is notable that the difference of the error of 96 modes and 6 modes is only 1%, despite that the force curves have visible differences at peak locations and amplitudes.

\[ \text{NF}_i = F_i + \sum_{j=0}^{N} F_{ij} \]  
(15)

where \( \text{NF} \) means interaction forces.

Fig. 5c plots the thrust coefficients of the DNS result and reconstructed forces with different numbers of modes in Region R2. RMS of thrust with 96 modes is 2.04, which is 2.5% larger than the DNS result. Decreasing the number of POD modes from 96 to 12 does not change the RMS error. The error of 6 modes slightly decreases to 2.0%, which is even smaller than the error of 96 modes. The result of 2 modes is -17.1%, approximately the same as the one with Region R1. However, the timing of peaks and valleys of 2 modes of Region R1 is different to that of Region R2. This is because the POD modes 1 and 2 and their temporal coefficients are different with respect to different sizes of computational domain.

In general, from these observations, it concludes that a limited number of POD modes can be sufficient to reconstruct the original force and POD modes that associate with higher energy are more important than those with lower energy in force reconstruction. The reconstructed force approaches to the solution of the impulse equation as the number of POD modes increases. However, results of the impulse equation may differ from DNS results, as we have shown in the prior validation section. The error of the POD-FSM caused by using less amounts of POD modes may compensate the error of the impulse equation. Therefore, statistic force calculated by several modes can be more approximate to DNS results.

Eq. (5) contains non-interaction and interaction force terms. The non-interaction term can be definitely accounted for a specific POD mode. However, it is difficult to distribute the interaction terms, which are caused by interactions of two modes, into one mode. Here, to make a comparison of the force of POD modes, we define the force related to the \( i \)th POD mode as

\[ \text{NF}_i = F_i + \sum_{j=0}^{N} F_{ij} \]  
(15)

where \( \text{NF} \) means interaction forces.

The interaction force history of the mean flow \( \text{NF}_0 \) and modes 1 \( \text{NF}_1 \) are shown in Fig. 6. For the force of the mean flow using Regions R1 or R2, the lift changes slightly as the number of modes change. However, the thrust with 2 modes significantly deviates from other curves. This implies that modes 3 and 4 play an important role in the thrust producing. From Fig. 6b, it can be seen that the lift curves with different mode number coincide for mode 1. This means that the lift of mode 1 is independent to the number of modes. It is notable that the trend of \( \text{NF}_0 \) and \( \text{NF}_1 \) do not resemble the DNS result. Particularly, the lift peak of \( \text{NF}_1 \) at approximately \( t=5.6 \) is much larger than the DNS result. Because \( \text{NF}_0 \) at \( t=5.6 \) is negative, adding \( \text{NF}_1 \) and \( \text{NF}_0 \) produces a value smaller than \( \text{NF}_1 \).

Comparing Fig. 6a and 6c, it seems that there is a phase shifting between the curves of Regions R1 and R2. Additionally, the amplitude of peaks and valleys is different. Because other flow parameters are the same, it can be concluded that the force calculated by the POD-FSM is not domain independent. This
differs from the impulse equation, by which the force calculated with different computational domains can be highly similar.

Figure 6. Interaction force of the mean flow and POD modes 1 of $St=0.6$ with different number of POD modes in Regions R1 and R2.

IV. Conclusion

A POD Mode Force Survey Method (POD-FSM) is presented to measure virtual forces of POD modes of a pitching-plunging plate. It shows that aerodynamic forces acting on the plate can be decomposed into a linear combination of the virtual force of each individual POD modes, the interaction between the POD modes, and the interaction between the POD modes and the plate. Several aspects of the POD-FSM, such as the effect of the number of POD modes and the sensitivity of computational domain, are examined. It is found that the superposition of the virtual forces of full POD modes can accurately predict the force acting on the plate within $\pm 6\%$ RMS errors for the flows in a range of $0.4 \leq St \leq 0.6$. Six high-energy contained POD modes can obtain a good approximation of the thrust within $\pm 5\%$ RMS errors. Two energetic POD modes can approximate the thrust with $\pm 18\%$ RMS errors. This indicates that POD modes with higher energy are more important than those with lower energy from the perspective of aerodynamic force. It is also found that the virtual force of each individual POD mode is domain dependent. Different sizes of computational domains produce different virtual force curves for POD modes.

Acknowledgments

This is work is supported under AFRL FA9550-11-1-0058 monitored by Dr. Douglass Smith and NSF CBET-1313217.

References

7. Wei, M. and Yang, T. A global approach for reduced-order models of flapping flexible wings. AIAA paper 2010-5085.


