Motion Kinematics vs. Angle of Attack Effects in High-Frequency Airfoil Pitch/Plunge

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Sinusoidal, “trapezoidal” and “triangular” pitch, all with identical nominal angle of attack limits and reduced-frequency of oscillation are compared mutually and with a related sinusoidal plunge case, in a combined experimental-computational study. Experiments (particle image velocimetry and dye injection) were conducted in a water tunnel at Reynolds number 10,000 with a SD7003 airfoil spanning the test section, while computations were with a 2D immersed-boundary method at Reynolds numbers of 300 to 1200. Of particular interest is testing of the assertion that reduced frequency, Reynolds number and angle of attack limits are the governing parameters for aerodynamic loads time history and evolution of vortex shedding, especially at the airfoil leading edge. This turns out to not be the case. Comparing a sinusoidal and a trapezoidal pitch, the latter shows a pairing of trailing shed vortices at every half-stroke, and a stronger leading edge vortex. Linear ramp or “triangular” pitch exhibits similar wake structures to those of the sinusoid, but leading edge vorticity is less pronounced. And in comparing sinusoidal pitch and plunge, similar leading edge vortex shedding is found if the plunge amplitude is defined by the airfoil leading edge displacement from the pitch case, wherein the quasi-steady effective angle of attack in plunge becomes almost twice that of pitch. All four cases had markedly different lift coefficient time history, though the sinusoidal pitch and plunge were reasonably close.

Nomenclature

\[ C_L \] = airfoil lift coefficient per unit span
\[ c \] = airfoil chord, =152.4mm
\[ f \] = airfoil oscillation pitch/plunge frequency
\[ \omega \] = pitch/plunge circular frequency
\[ h_0 \] = non-dimensional plunge amplitude, normalized by \(c \)
\[ h \] = plunge position as function of time
\[ k \] = reduced frequency of pitch or plunge, \( k = \alpha_c / 2U_0 \)
\[ St \] = Strouhal number, \( 0.02/2 / fch_0 \)
\[ U_\infty \] = freestream (reference) velocity
\[ Re \] = Reynolds number, \( Re = c U_\infty / \nu \), \( \nu \) taken as \(10^{-6} \) in SI units for water at 20\(^\circ\)C
\[ \alpha \] = kinematic angle of incidence due to pitch
\[ A \] = pitch amplitude, in degrees
\[ \alpha_0 \] = mean angle of attack (that is, the constant incidence angle offset from zero), = 4\(^\circ\)
\[ \alpha_c \] = effective total angle of attack in pitch or plunge
\[ x_p \] = pitch pivot point: fraction of chord downstream from airfoil leading edge
\[ t/T \] = dimensionless time, in fractions of one oscillation period
\[ \phi \] = phase difference between pitching and plunging; positive \( \rightarrow \) pitch leads

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Introduction

Examples of literature in low-Reynolds number pitching and plunging airfoils\cite{1,2,3}, and dynamic stall\cite{4} give evidence that aerodynamic loads time history and vortex shedding for a broad class of unsteady airfoil problems depend mainly on (1) the amplitude of the angle of attack variation, (2) the rate of motion and (3) the mean angle of attack. The reduced frequency and Reynolds number range is relevant to prototypes in nature (hummingbirds, dragonflies, and so forth) and to the flight vehicles inspired by such prototypes. For the latter, there are two applications of interest: flapping-wing aerodynamics, and gust response for all vehicle classes, whether fixed-wing, rotary-wing or flapping-wing. In wind tunnel testing typically a pure-pitch sinusoidal oscillation about the airfoil quarter chord is the motion of choice\cite{4}. This is straightforward to implement at the high physical rates necessary to obtain large reduced frequencies in wind tunnels, and is most relevant to rotorcraft applications or to fixed-wing aircraft maneuvering. The less common alternative is pure-plunge. But the actual type of motion – whether pure-pitch, pure-plunge or a different and possibly non-trigonometric combination, is of secondary importance, provided that the above-mentioned conditions (1)-(3) match in each case.

But for high motion rates and low Reynolds numbers there remains the question of the dependency of the aerodynamic loads and the flowfield time evolution on the actual motion time history, beyond merely matching (1)-(3). One example is skewed sinusoidal time trace of pitch angle of attack\cite{5}. An alternative approach, pursued here, is to consider an amalgamation between ramp-type motions and sinusoidal periodic motions, where the motion has sharp corners and linear segments of velocity, but remains periodic. The idea is that discontinuities in acceleration – or at least, practically realizable approximations thereto – may result in significant departures from sinusoidal motions with identical reduced frequency and extremes of angle of attack, but where position time history infinitely differentiable. As a secondary objective we consider similarities in leading edge vortex shedding between pure-pitch and pure-plunge oscillations. Such comparisons are not new to the dynamic-stall literature (for example, Fukushima and Dadone\cite{6}), but the subject can be extended to Micro Air Vehicle applications with more detailed information on shed vorticity, enabled by ease of flow visualization in water at high reduced frequencies.

We present a sequence of experiments at the same reduced frequency and angle of attack amplitude, but differing in kinematics:

1. Sinusoidal pitch, pivoting about \( x_p = 0.25 \), \( \alpha(t) = \alpha_0 + A \cos(2\pi ft + \varphi) \).
2. Trapezoidal pitch, \( x_p = 0.25 \).
3. Linear-ramp or triangular pitch, \( x_p = 0.25 \).
4. Sinusoidal plunge, \( h(t) = h_0 \cos(2\pi ft) \), \( h_0 = 0.092 \), with offset angle \( \alpha_0 = 4^\circ \).

The choice of total angle of attack amplitude (21.5\(^\circ\)) continues prior work\cite{8}, and was chosen to be large enough to place the airfoil in deep stall, from the static point of view. The geometry of airfoil oscillation is given in Figure 1. All pitch motions commence with the airfoil at

![Figure 1. Schematic of airfoil pitch and plunge oscillation](image-url)

its maximum angle of incidence (25.5\(^\circ\)), for continuity of pitch speed from rest. Thus the first half of the first period of motion can be considered in isolation as a return-from-stall ramp motion.
The above cases are studied experimentally and computationally. Particle image velocimetry results for Case 1 are discussed in McGowan et al\textsuperscript{7}. In the present experiments, Re = 10,000; in the computations, Re ranges from 300 to 1200. The discrepancy in Re is nontrivial, but is unavoidable given the respective limits of the two approaches. In the water tunnel a much lower Re requires a flow speed too low for stable operation and good flow quality, the computational resource limits prohibit a much larger Re. The plunge case, motivated by pitch-plunge comparison begun in McGowan et al\textsuperscript{7}, has a plunge amplitude equal to the displacement of the airfoil leading edge in the pitch cases. The assertion is that displacement of the leading edge, and not matching of quasi-steady angle of attack between pitch and plunge, is the main parameter governing the strength of leading edge vortex shedding and lift coefficient time-trace.

**Experimental Setup**

Experiments used the High-Intensity Pitch-Plunge Oscillator rig in the Horizontal Free-surface Water tunnel at the Air Force Research Lab, Air Vehicles Directorate. The water tunnel approach allows high reduced frequencies of oscillation despite moderate physical rates, and therefore high motion accuracy and comparatively simple data acquisition. The oscillator rig consists of two linear electric motions, each independently actuating one vertical “plunge rod”, thus enabling the two degrees of freedom. The plunge rods ride inside a vertical shroud that pierces the free-surface. The upstream rod is constrained in Teflon bushings to translate purely vertically, while the downstream rod is securing to its linear motor via a pivot, and is allowed to swing inside its bushings, allowing for ±45° of airfoil pitch in the current implementation, with almost arbitrary choice of pitch pivot point. The rig (Figure 2, left-hand side) is described in more detail by Ol\textsuperscript{8}.

![Figure 2. Pitch/plunge oscillator rig: schematic of full rig (left) and photograph of SD7003 c = 152.4 mm model installed in the water tunnel test section.](image)

The SD7003 model with 18” (457 mm) span and 6” (152.4 mm) chord is constructed from 304-series stainless steel. The model was cut by wire-EDM from 4”-thick blocks subsequently welded together, to a contour thickness of 0.035” (0.89 mm), as a compromise between minimizing model mass and maintaining surface geometric fidelity. The model connects to the rig’s plunge rods by a central mount on its pressure side (Figure 2, right-hand side). The upstream rod pivots at the model x/c =0.25 location, while the downstream pivots at x/c = 0.5408.

In each case the motion was smoothed by applying an acceleration upper-bound of 10 m/s\textsuperscript{2}. Following prior work\textsuperscript{8} and motivated by the desire for nonzero mean lift coefficient, oscillations in all cases are about an offset value of $\alpha_0 = 4^\circ$.  

Measurements consisted of particle image velocimetry (PIV) and traditional dye injection with food-coloring blue ink, from a 0.5mm-diameter probe placed on the pressure side of the airfoil, at x/c ~ 0.02 aft of the leading edge. The dye was driven by a positive-displacement pump, matching dye exit velocity to the nominal free-stream velocity. The PIV uses a PCO-4000 11 M-pixel camera, and algorithms based on Willert and Gharib, with implementation described by OI. Typical PIV resolution was 160 pix/cm, or about 150 32x32 pixel PIV windows per chord, with 50% overlap.

**Computational Setup**

The incompressible Navier-Stokes equations are solved by a finite-difference-based Cartesian-grid immersed boundary solver, allowing for simulation of flows with complex immersed 3-D moving bodies; the present implementation however is 2D. The key feature of this method is that simulations with complex boundaries can be carried out on stationary non-body-conformal Cartesian grids, eliminating the need for complicated remeshing algorithms that are usually employed with conventional Lagrangian body-conformal methods. The method employs a second-order central difference scheme in space and a second-order accurate fractional-step method for time advancement. The Eulerian form of the Navier-Stokes equations is discretized on a Cartesian mesh and boundary conditions on the immersed boundary are imposed through a “ghost-cell” procedure. The pressure Poisson equation is solved using the geometric multi-grid method integrating with immersed-boundary methodology.

**Results**

Idealized time-traces of the pitch angle of attack for sinusoidal, trapezoidal and linear-ramp motions are shown in Figure 3, including the phases of motion at which PIV and/or dye-injection data are presented; values of geometric incidence for pitch are with respect to the left-side vertical axis. The choice of phases at which to take data is motivated by the trapezoidal pitch, where phases “a”, “c”, “e” and “g” are at the vertices of the trapezoid, while “b” and “f” are halfway on the downgoing and upgoing strokes, respectively.

![Figure 3. Sinusoidal (green), trapezoidal (black) and triangular (blue) time traces of pitch angle; and sinusoidal plunge-induce angle of attack (orange). “a” – “h” mark phases where data were taken.](image)

For the sinusoidal plunge, the quasi-steady motion-induced angle of attack is shown as the orange curve in Figure 3, denoted with respect to the right-hand vertical axis. The quasi-steady motion-induced angle of attack amplitude with $h_0=0.092$ is $\alpha_e = -\arctan(h_0 \dot{U}_x) = 36.0^\circ$; vs. $21.5^\circ$
pitch geometric incidence amplitude. $h_0 = 0.092$ is equal to the airfoil leading edge peak displacement with 21.5° pitch and $x_p = 0.25$. It is 90° out of phase with the pitch motions, as the plunge motion begins with the airfoil at rest at the top of the stroke; the angles of attack are with respect to the right-side vertical axis. The phase difference between pitch and plunge has a significant effect on the startup conditions and relaxation to time-periodicity, but not on the time-periodic flow near the leading edge, as discussed below. Also, it should be noted that the time trace of angle of attack for plunge with sinusoidal variation of elevation is itself not strictly sinusoidal.

The triangular motion or linear ramp simply connects peak positive and negative pitch angles in the sinusoid, while the trapezoidal motion matches the maximum positive and negative pitch rates of the sinusoid. This matching then defines the duration of position-hold at the top and bottom of the stroke. The motivation of the trapezoidal motion is that by matching the sinusoid’s angle of attack limits and peak angle of attack rates, the principal terms of the circulatory lift response in quasi-steady airfoil theory look identical – suggesting that flowfield velocity and vorticity time history should look similar, if linear concepts remain at least notionally valid. The triangular pitch, on the other hand, has a lower pitch rate of nominally constant magnitude, with an “instantaneous” switch of direction at upper and lower extremes of angle of attack.

1. Sinusoidal, Trapezoidal and Linear-Ramp Pure Pitch: Start-up Transients

We first report, using dye injection, the evolution from startup for the three pitch cases: trapezoidal (Figure 4 and continued in Figure 5), sinusoidal (Figure 6 and continued in Figure 7, and linear ramp or “triangular” (Figure 8 and continued in Figure 9). In an effort to minimize the number of pictures while adequately resolving salient features, formatting of these figures is as follows: the trapezoidal and sinusoidal pitch cases are shown in phases “a”, “b”, “c”, “e”, “f”, and “g”. Phase “d” is similar to “c” and “h” is similar to “g”, especially for the trapezoidal case, where theoretically there is no motion between those respective phases. The triangular pitch case is shown in phases “a”, “b”, “d”, “e”, “f” and “h”, retaining the extremes of motion at “d” and “h”. All three pitch cases are shown across eight periods of oscillation, from motion onset until past the stage where the flow has relaxed into phase-periodicity.

As the main objective was to observe leading-edge vortex shedding and subsequent LEV evolution, dye was injected near the leading edge. Trailing edge dye injection, which is the more usual approach\(^1\), produces a more aesthetically pleasing rendition of the wake, with higher dye concentration and therefore higher contrast. Nevertheless, the qualitative wake structure is discernable even with leading-edge injection.

The apparent strength of the LEV and clutter of low-contrast dye on the suction side of the airfoil is partially attributable to slight lifting of the dye probe from the airfoil surface. The exact chordwise position of the dye injection probe also has an effect: the closer to the airfoil leading edge, the more apparently coherent the leading edge vortex. Neither cause however makes any discernable difference in the wake aft of the trailing edge or in broad conclusions about the LEV.

For all of the pitch cases, commencement of oscillation from a deep-stall incidence angle requires 4-5 periods to reach time-periodicity; that is, where snapshots of the n+1\(^{th}\) cycle are not substantively distinguishable from the nth. The physical period of oscillation is 1.846s, or 0.8 convective times. Thus, relaxation to periodicity takes as many as 4 convective times. This contrasts with lower-amplitude (though in some cases higher frequency) cases discussed by Ol8, where 1.5-2 cycles were sufficient to attain periodicity.

The two motions with discontinuity in angle of attack rate – trapezoid and triangle – evince a concentrated vortex shedding just downstream of the trailing edge, in phase a and especially phase f. This is most pronounced for the triangle, which has the largest alpha rate discontinuity. The sinusoid has a more ambiguous TE vortex formation, and a strong vortex is not evident until phases b and f, where pitch rate is maximum.
Figure 4. 8 periods of trapezoidal pitch: dye injection at phases a (left column), b (middle column) and c (right column); time-evolution from motion onset is from top to bottom. Top row is first period, second row is second period, and so forth, down to the 8th period.
Figure 5. 8 periods of trapezoidal pitch, continued: sampling at phases e, f and g.
Figure 6. Sinusoidal pitch dye injection: phases a (left column), b (middle column) and c (right column). Top row is 1\textsuperscript{st} period, 2\textsuperscript{nd} row is 2\textsuperscript{nd} period, ...., bottom row is 8\textsuperscript{th} period.
Figure 7. Sinusoidal pitch dye injection, continued: phases e, f and g.
Figure 8. Triangular (linear ramp) pitch dye injection: phases a, b and d.
Figure 9. Triangular (linear ramp) pitch dye injection, continued: phases e, f and h.
The principal distinction between the trapezoidal pitch the other two cases is the former’s
double-formation of shed vortices twice per stroke. This is clear in the dye injection images in
phase “f” (circled in red in period-8, phase-f of Figure 5), and to a lesser extent in phase “b”
circled in red in period-8, phase-b of Figure 4. The greater clarity of vortex-doubling in phase f vs.
b is ascribed to the camber of the airfoil and positive offset, $\alpha_0 = 4^\circ$. Vortex-doubling is apparent
as early as the second period, at least for phase f.

2. Sinusoidal, Trapezoidal and Linear-Ramp Pure Pitch: Periodic

Figure 10. Trapezoidal pitch: phase-averaged PIV at Re = 10,000 (left column) and
instantaneous CFD at Re = 1200 (right column) vorticity contours: first 4 of 8 phases.
We next consider the three pitch cases after the flowfield response has relaxed to time-periodicity. PIV data were taken in sequences of 120 image pairs per motion phase (“a” through “h”), with the first 5 pairs disregarded and the remaining 115 ensemble-averaged for each phase. The computation, on the other hand, was run for 6 periods from motion onset. The 6th period is considered by itself, with no phase-averaging.

For the trapezoidal-pitch case, the out-of-plane vorticity contours are shown in Figure 10 and Figure 11, together with computed results at Re = 1200. Experimental results for the sinusoidal pitch case are repeated from an earlier experiment, reported in McGowan et al. These are compared with computation, with the latter again at Re = 1200, in Figure 12. Only four phases

Figure 11. Trapezoidal pitch: PIV at Re = 10,000 (left column) and CFD at Re = 1200 (right column) vorticity contours: second 4 of 8 phases.
of motion were available for this data set. For the linear-ramp pitch, PIV data are not available, and only the Re = 1200 computation is shown, in Figure 13. In all cases vorticity is normalized by c and \( U_\infty \). Contour levels are from -36 to +36, with the two levels about zero blanked-out for clarity.

The aforementioned vortex doubling for the trapezoidal pitch is also clear from the phase-averages in the respective phases in the PIV and computation (Figure 6 and Figure 7), suggesting that the phenomenon is strongly periodic. The computation is expected to be more periodic than the experiment, and indeed it’s evidence of vortex pairing is even stronger. Evidently, the two

![Figure 12. Sinusoidal pitch: PIV at Re = 10,000 (left column) and CFD at Re = 1200 (right column) vorticity contours at 4 phases of motion.](image)
“shoulders” at the angle of attack extremes of the trapezoid are responsible for a shedding akin to that of a starting-vortex in impulse start.

All of the pitch cases evince a LEV forming at or near the top of the pitch stroke. From the dye injection results, the strength of this LEV is somewhat greater in the trapezoidal than the sinusoidal, and in turn appreciably greater than for the triangular pitch. The triangular pitch’s weaker LEV is intuitively attributable to a lower peak pitch rate. More properly, for the trapezoidal pitch one should speak of a LEV pair rather than a single vortex, albeit the positive-signed concentration of vorticity is much weaker than the negative. For the trapezoid both the PIV and computation show contours of positive and negative vorticity shed from the LE. Dye injection of course cannot distinguish sign of vorticity.

Whereas the dye injection points clearly to the triangular pitch having the weakest LEV system, the computation suggests that the sinusoidal pitch’s LEV is weaker. We speculate that the “vertex” of the triangular angle of attack time history has an effect akin to the trapezoidal pitch’s “shoulders”, where a discontinuity in angle of attack rate promotes LEV shedding in the computation. In any case, in the experiment it is reasonable to surmise that for the trapezoid and

Figure 13. “Triangular” or linear-ramp pitch: computed vorticity contours, Re = 1200, 8 phases of motion.
sinusoid the LEV remains sufficiently coherent to interact with the trailing vortex system by the
time that it arrives at the trailing edge. The triangular pitch’s LEV, being weaker, does not have
such an obvious relationship with the TE vortex system. In the computation the trapezoid is the
only case for which this presumed LEV-TEV interaction is substantiated.

3. Reynolds Number Effects in the Computation

Figure 14 repeats the computation for the trapezoidal-pitch case, at Re = 300, 600 and
1200, for the six phases “a”-“e” and “e-g”. Clearly, increasing Re causes decreasing dissipation
and therefore less apparent attenuation of the LEV as it progresses downstream – and therefore
stronger interaction between the LEV when it arrives at the airfoil trailing edge, and with the
vortices shed native to the trailing edge vicinity. At Re = 300 there is at most a vague
concentration of vorticity formed at the leading edge, convecting downstream adjacent to the
airfoil’s suction. At Re = 600 there is a discernable LEV ejected from the airfoil surface, and at Re

Figure 14. Computed vorticity contours for trapezoidal pure-pitch: Re = 300 (left column), 600
(middle) and 1200 (right) at six phases of motion.
= 1200 a leading-edge vortex pair, wherein the positive-vorticity is due to usual dynamic-stall situation – that is, reverse (upstream) flow between the LEV and its leading-edge feeding sheet. At Re = 1200 there is also some indication of instabilities forming in the negative-vorticity (blue) feeding sheet at the trailing edge, for example at phase “c” and “e”. These instabilities lead to discretized concentrations of one sign of vorticity in the feeding sheets in the PIV results in Figure 10 through Figure 12. It is of course also unavoidable that boundary layers – as evinces by wall-bound sheathes of vorticity of one sign – are much thicker for the Reynolds numbers of the computation than for the experiment; indeed, at Re = 100 the boundary layer concept is itself suspect.

4. Sinusoidal Pitch vs. Sinusoidal Plunge

Pitch-plunge comparison is useful in the context of extending quasi-steady concepts. Perhaps through deeper understanding of pitch and plunge as canonical motions, it will be possible to “explain” all 2DOF airfoil oscillations, in the sense of pitch and plunge spanning the space of all possible 2DOF oscillations. While it remains premature to justify such lofty ambition, a preliminary extension relevant to massively-separated cases is assessment of what plunge amplitude is required to produce LEV strength similar to that observed in pitch. Matching plunge-induced and pitch-geometric angle of attack was shown to fail to give matching leading edge separation, but matching displacement of the airfoil leading edge between the pitch and plunge oscillations shows promise. As noted in Figure 3, the resulting induced angle of attack in plunge is far larger than the geometric angle of attack in pitch, nor does sinusoidal variation of plunge position produce truly sinusoidal angle of attack variation at these high motion rates.

Dye injection results for 5 periods after startup for the plunge case are given in Figure 15 for phases “a”, “b” and “d”, and continued in Figure 16 for phases “e”, “f” and “h”. Not

Figure 15. “Sinusoidal” plunge dye injection, phases a, b and d.
surprisingly, because the motion commences from $\alpha_0 = 4^\circ$ and not from deep-stall, the relaxation to time-periodicity takes fewer periods of oscillation than reported for the pitch cases above; by the fourth period, if not sooner, the flow has reached periodicity.

Comparing phase “d”, which corresponds loosely to phase “b” of the pitch cases (referring to the angle of attack time traces in Figure 3), it is evident that the LEV is in fact very similar to that of the sinusoidal pitch, as shown in Figure 6. This favorable comparison holds for all phases where the LEV has not convected far from the leading edge, and breaks down as the LEV progresses further downstream. The wakes behind the trailing edge are in fact entirely different between the plunge and any of the pitch cases. For the plunge, the wake is apparently a reverse-Karman street with positive and negative concentrations of vorticity convecting downstream at $U_\infty$. This is confirmed, at least qualitatively, by the computation in Figure 17; PIV results were not available. As for the pitch cases, the computation is from the 6th period of a 6-period sequence starting from rest. All 8 phases, a-h, are shown.

Similarly to trapezoidal pitch, but unlike sinusoidal pitch, the computation’s resolution of LEV formation and downstream progress compares well with the dye injection results. This is consistent with the observation by McGowan et al., that PIV-CFD comparison is more successful for plunge than for pitch. The dynamic stall vortex, or what might be termed as such, forms near phase “b” and is clearly visible by phase “c”, together with the revise-signed concentration of vorticity erupting from the boundary layer between the LEV and its feeding sheet. By phase “f”, the positive-signed concentration of vorticity has dissipated, and by phase “h” the LEV has lifted off the airfoil suction-side. Continuing one full period of motion later, again at phase “f” one can see that the previous cycle’s LEV has made its downstream journey all the way to the trailing edge,

Figure 16. Sinusoidal plunge dye injection continued, phases e, f and h.
merging with the TE-shed vortex system albeit not, arguably, affecting it much. Qualitatively the behavior of the LEV from its onset through at least one period of oscillation is corroborated by the computed vorticity contours in Figure 16. LEV shedding is resolved well, but computation does not resolve the aforementioned interaction near the trailing edge. In the wake, feeding sheet connecting vortices of opposite sign is not resolved, but the wake structure in terms of position of vortices is a good match with the dye injection.

![Computed sinusoidal-plunge vorticity contours; phases a-h, Re = 1200](image)

The idea of LEV downstream convection and interaction with the TE vortex system resembles the “shear layer vortex” of McAlister and Carr\(^1\), and Walker et al\(^2\). However, the shear layer vortex and LEV (or more properly, dynamic stall vortex) form essentially simultaneously, whereas in the present work we find that the vorticity concentration on the airfoil suction side just upstream of the TE is the leading edge vortex from a prior cycle of oscillation.

5. Lift Coefficient Time History.

It is fitting to ask to what extent differences in the vorticity contours and dye injection images between the four cases would relate to differences in time-traces of integrated aerodynamic loads. As a force balance in the HFWT is not yet available, only computational results for lift coefficient, \(C_L(t/T)\), are presented, for \(Re = 1200\), for \(5 \leq t/T \leq 6\), in Figure 18. Lift coefficient is normalized as usual – by dynamic pressure based on \(U_\infty\) and the model chord, \(c\).

Peak \(C_L\) magnitude is on the order of 10. This grates the intuition when viewed from the point of quasi-steady aerodynamics. However, the apparent-mass contribution for a flat-plate oscillating at the \(k\) and \(A\) of the present work gives a max normal-force coefficient also on the order of 10, suggesting that the bulk of the lift response is “noncirculatory”, in the sense of not being associable with bound vorticity in the quasi-steady sense. In all of the nonsinusoidal motions, the computation returns a nonphysical load spike at every point of discontinuity of angle of attack rate – at the “vertices” of the triangular pitch, and “shoulders” of the trapezoidal pitch. There are also spurious high-frequency oscillations which are shown here without filtering. Nevertheless, these integrate to an arguably unimportant contribution, and the broad trends comport with qualitative predictions of dye injection and PIV:
- the sinusoidal pitch and sinusoidal plunge have similar $C_L$ amplitude, and the phase difference between the two is follows the phase difference in their respective angle of attack time history. For both, the $C_L$ response is essentially sinusoidal.
- trends in lift coefficient magnitude follow trends in apparent size of LEV: greatest for the trapezoidal pitch, intermediate for the sinusoidal, and least for the triangular pitch.
- Lift response is, loosely speaking, related to both angle of attack and angle of attack rate. This combination suggests why lift for the trapezoidal and triangular pitch is not trapezoidal and triangular, respectively – but that a jump in CL occurs at every $t/T$ value where the angle of attack rate is discontinuous. The rate-effect can also be blamed for the spikes in lift coefficient at points of $\alpha(t/T)$ discontinuity.

Fine features in the flowfield, such as the double-vortex shedding of the triangular pitch at phases “b” and “f”, are not clearly associable with any feature in the computed $C_L(t/T)$ time history. Differences between the four motions are nevertheless qualitatively very clear, and are at least as severe as those in the flowfield structures. Of course, deeper analysis awaits direct force measurement in the water tunnel.

Figure 18. Computed $C_L(t/T)$ for the three pitch cases and plunge case, $Re = 1200$, over one period of motion, when time-periodicity has been achieved.

**Conclusion**

Investigation of Low Reynolds number (10,000 in experiments, 300-1200 in computations) 2D airfoil sinusoidal plunge, and sinusoidal, trapezoidal and linear-ramp pitch, shows that matching of angle of attack limits, reduced frequency and Reynolds number is insufficient to obtain the same wake geometry or loads time history. The implication is the need for a more general criterion of angle of attack time history, with consequences for the flight dynamics of maneuvering Micro Air Vehicles. Trapezoidal pitch has a double-vortex structure in the near wake, associable with the pitch incidence angle plateau of the trapezoid. While the angle of attack time history between trapezoidal and sinusoidal pitch is closer than to that of the triangular or linear-ramp pitch, the sinusoidal and triangular ramps have more similar wake
structure. The trapezoidal case is also the outlier in that computation-experimental agreement was quite good for that case, and poor for the other two pitch cases.

All of the cases examined here have leading edge vortex shedding. The LEV convects downstream and interacts with the trailing-edge vortex structure. The triangular pitch’s LEV is weakest, followed by the sinusoidal and then trapezoidal pitch. The sinusoidal plunge matched the LEV of the sinusoidal pitch fairly well, but in so doing its quasi-steady induced angle of attack was almost double the geometric incidence angle of the pitch, and the wake of the plunge case was markedly different from that of any of the pitch cases.

Qualitatively the case with the largest LEV also has the highest amplitude of lift coefficient. This will be investigated further in future work, where the water tunnel experiments will be extended to include direct force measurements, while the computation will try to approach the Reynolds number of the experiment.

References