Effects of Unequal Pitch and Plunge Airfoil Motion Frequency on Aerodynamic Response

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Departing from mainstream pitching/plunging airfoil literature, we consider scenarios where the frequencies of the plunge and the pitch are not equal. The motivation is, first, consideration of flows with very large excursions in total angle of attack, for assessment of motion time history effects; and second, an attempt to model the gust response of flapping-wing Micro Air Vehicles by considering the higher-frequency motion as the prototype kinematics for the flapping, and the lower-frequency motion as prototype for the gust. Computations with immersed boundary methods at Re < 1000 are compared with water tunnel experiments at Re = 10,000. Despite the enormous difference in Reynolds number, qualitative behavior of vortex shedding was found to be very similar, with implications of broadening the scope of very low-Re simulations for such flows in general.

Nomenclature

\( h \) Plunge (heave) as function of time
\( h_1 \) Amplitude of plunge
\( h_0 \) Reduced plunge amplitude, \( h_0 = A/c \)
\( c \) Airfoil chord
\( f \) Pitch or plunge physical frequency, Hz
\( T \) Plunge motion period (seconds)
\( t/T \) fraction of plunge motion period
\( k \) Reduced pitch or plunge frequency, \( k = \pi fc/U_\infty \)
\( \alpha \) Amplitude of pitching motion, deg
\( \alpha_p \) Angle of attack due to pitch, as function of time
\( \alpha_T \) Total effective angle of attack, due to combined pitch and plunge, evaluated at \( x/c = 0.25 \)
\( \alpha_0 \) Mean geometric pitch angle
\( x/c \) Airfoil chord fraction, or chord fraction of pivot point location for pitch (trailing edge = 1.0).
\( St \) Strouhal Number, \( St = 2fl/U_\infty = 2kh/\pi \)
\( \varphi \) Phase of motion, deg; 360° = one period of plunge
\( C_T \) Thrust (or drag) coefficient
\( C_L \) Lift coefficient

I. Introduction

The unsteady aerodynamics of pitching and plunging airfoils has principally two traditional motivating applications: one is the engineering application of dynamic stall of aircraft lifting surfaces in aggressive maneuver, and of helicopter blades. The other is the aerodynamics of flapping wings for swimming and flying animals. In both cases abstractions to sinusoidal pitching and plunging of nominally two dimensional airfoils have been useful for studying the evolution of vortex shedding, the time dependency of aerodynamic loads, and so forth. But, in essentially all cases the pitch and plunge frequency is taken to be equal, as this is the representative scenario for both engineering and biological applications, as well as the classic problem in unsteady airfoil aerodynamics. In the

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The present work we consider a generalization of two dimensional airfoil pitch and plunge, with relevance to Micro Air Vehicle (MAV) applications.

Typically, the pitch sinusoid leading the plunge by a phase difference of 90° is close to “optimal” for thrust efficiency. Induced angle of attack due to plunge partially cancels the geometric angle of attack from pitch, so that the total instantaneous angle of attack remains modest even for large stroke amplitudes and frequencies. In the present work, we retain this phase difference, but consider situations where the pitch frequency and plunge frequency differ by multiples other than one. At some phases of the motion, the pitch contribution to angle of attack will oppose the plunge contribution, while in other phases it adds constructively, resulting in very large excursions in angle of attack. While the resulting kinematics is unlikely to be useful for modeling either flapping-wing animals or for applications in engineering, it is a useful ploy for studying the massive vortex shedding resulting from such angle of attack variations, and more importantly, for further pursuit of the longstanding question of the extent to which the loads time history comports with the angle of attack time history.

Another application of these “mixed frequency” cases is a crude model of gust response for flapping wings, where the fast frequency (pitch) is a single degree of freedom model for flapping, while the slow frequency (plunge) is intended to model a sinusoidal gust. Such an approach is useful in wind or water tunnels where shuttering the tunnel is not possible, and the model’s oscillations are directly modulated as an alternative approach to measuring gust effects.

In this paper the Reynolds number of the experiment greatly exceeds that of the computation, for reasons of limitations of both. One of our aims is to consider the extent to which such large discrepancies in Reynolds number may nevertheless afford reasonable experimental-computational comparison, for problems with massive unsteadiness. An answer in the affirmative would place on firmer ground computations where the Reynolds number is artificially low.

II. Methodology

A. Airfoil Motion Parameters

The motion of an airfoil undergoing combined pitch-and-heave can be prescribed by:

\[ h = h_1 \sin(2\pi f_h t + \phi); \]
\[ \alpha = \alpha_o + \alpha_p \sin(2\pi f_p t); \]

where \( h_1 \) is the heave amplitude, \( \alpha_o \) is the pitch-bias angle and \( \alpha_p \) is the amplitude of the sinusoidal pitch angle variation, \( f_h \) is the plunge frequency, \( f_p \) is the plunge frequency, and \( \phi \) is the phase difference between the pitch and plunge motions. Pitch-plunge frequency ratio is defined as: \( FR = f_p / f_h \). In addition to \( \alpha_o, \alpha_p \) and \( \phi \), the relevant non-dimensional parameters are the normalized plunge amplitude \( h_1^* = h_1 / c \), the Reynolds number \( \text{Re}_x = U_x c / \nu \) (where \( U_x \) is the freestream velocity, \( c \) is the foil chord and \( \nu \) is the kinematics viscosity of the fluid) and the Strouhal number \( St = 2h_1 f_h / U_x \) based on the wake width \([6]\). An alternative frequency parameter is the reduced frequency, \( k = \pi f_h c / U_x \), here defined based on the plunge frequency. Note that \( kh_1^* = \pi St / 2 \), and this product is also equal to the peak plunge velocity normalized by the freestream velocity. The total effective angle of attack, \( \alpha_T \), is a combination of the pitch geometric angle of attack, the plunge-induced angle of attack, and an additional contribution when the pitch pivot point is not at x/c = 0.25 (as discussed by Ol’). That is, when the airfoil pitches about x/c ≠ 0.25, there is an effective “plunge” component at x/c = 0.25, which enters the \( \alpha \)-calculation when \( \alpha \) is evaluated at x/c = 0.25. We assume that the “correct” interpretation of angle of attack is the value at x/c = 0.25. Whether or not lift, for example, is in some way proportion to angle of attack will of course depend on the validity of our assumption.

Following prior experimental work\([2]\), we use the SD7003 airfoil\([1]\) in the present investigation. The heave amplitude, \( h_1 \), is fixed at a value of 0.1 and the pitching amplitude, \( \alpha_i \), is 15°, a mean pitch-bias angle, \( \alpha_0 \), of 4°. The phase angle, \( \phi_1 \), is 90°, with pitch leading plunge. We consider principally FR=2, and compare with the usual FR = 1. Also, the location of pivot point for the pitch is also varied. The common choice of pivot point, x/c = 0.25, is considered, as well as x/c = 0 (pivot about the leading edge), 0.5, 0.75 and 1.0 (trailing edge), with most focus on x/c = 0.5. Cases covered by computational and experimental approaches are listed in Table 1. We note the large discrepancy between the Re range of computations and experiment. For the experiment, the lower bound of Re is
determined by facility minimum flow speed and airfoil physical size. For the computation, the upper bound of \( Re \) is driven by the desire to avoid turbulence modeling or subgrid models, together with grid size limitations. \( Re = 300 \) is the baseline Reynolds number for the computations. The factor of \( \sim 30 \) ratio in experimental vs. computational Reynolds number presents a speculative but intriguing scenario of what can or can not be compared in the physics of the two respective problems.

### Table 1. Summary of cases for the computation (left) and experiment (right).
The experiment refers to dye injection runs; this single PIV run is shown with an asterisk.

<table>
<thead>
<tr>
<th>( Re )</th>
<th>Pivot point ((x/c))</th>
<th>Frequency Ratio (FR)</th>
<th>( Re )</th>
<th>Pivot point ((x/c))</th>
<th>Frequency Ratio (FR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0.5</td>
<td>2.0</td>
<td>10,000</td>
<td>0.0, 0.25, 0.5*, 0.75, 1.0</td>
<td>1.0, 2.0*</td>
</tr>
<tr>
<td>300</td>
<td>0.0, 0.25, 0.5, 0.75, 1.0</td>
<td>0.5, 1.0, 2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>0.5</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**B. Computational Method**

A second-order finite-difference-based immersed-boundary solver \(^{[19,20]}\) has been developed. Its key feature is that simulations with complex boundaries can be carried out on stationary non-body conformal Cartesian grids, eliminating the need for complicated re-meshing algorithms that are usually employed with conventional Lagrangian body-conformal methods. The Eulerian form of the Navier-Stokes equations is discretized on a Cartesian mesh and boundary conditions on the immersed boundary are imposed through a “ghost-cell” procedure \(^{[8,9]}\). The method also employs a second-order central difference scheme in space and a second-order accurate fractional-step method for time advancement. The pressure Poisson equation is solved using the geometric multi-grid method integrating with immersed-boundary methodology. Detailed validation of the code can be found in \([10]\). Based on comprehensive grid and domain size independence studies conducted for the motions studied in the present work, a domain size of \( 30 \times 30 \) and a \( 553 \times 321 \) grid has been chosen for all simulations.

A uniform free-stream is specified in the computation, and the airfoil begins moving at time \( t=0 \), or phase \( \varphi=0^\circ \); that is, it is not the case that the computation would run for several convective times solving the steady-state airfoil problem, and then the airfoil would start pitch/plunge. In the experiment, however, the airfoil is held fixed for many convective times before motion and data recording commence.

**C. Experimental Setup**

Experiments were conducted in the U.S. Air Force Research Laboratory’s Horizontal Free-surface Water Tunnel (“HFWT”); the HFWT and the approach used for airfoil PIV static measurements are described in Ol et al.\(^4\). For the present unsteady measurements, PCO 4000 11Mpix cameras\(^5\) were triggered off of an external pulse train derived from the position encoder of the motion rig, thus allowing for selection of motion phase at which to acquire data. Airfoil motion was driven by a 2-degree-of-freedom “pitch-plunge rig”, consisting of a pair of electric linear motors mounted vertically on a plate above the tunnel test section free-surface. Each motor actuates a vertical rod, which connects via a bushing to the airfoil at a fixed pivot point on the airfoil chord. Motion trajectory of each rod is programmed independently, allowing for single degree-of-freedom motions such as sinusoidal pure-pitch or pure-plunge, as well as nontrigonometric and combined motions. The pivot point for pitching motions can be varied as
well, but for simplicity was limited in the present study to the quarter chord. Details of the pitch-plunge rig are given in Ol.

All of the experimental data was collected at Re = 10,000, corresponding to ~ 6.7 cm/s flow speed in the PIV experiments. This is near the lower bound of tunnel operating velocity; at lower speeds, flow quality begins to degrade, possibly because the grid Reynolds number for the screens in the settling chamber becomes too low for the screens to function properly.

Since the plunge is a cosine wave, and the pitch is a negative sine wave, it is not possible to start both without discontinuity in velocity. Plunge was started smoothly, but pitch was “forced” to start from maximum pitch velocity, meaning that the motion controller was commanded to achieve full pitch velocity as quickly as possible. To achieve this without excessive parasitic oscillations in angle of attack requires selection of PID parameters for the motion controller on a case by case basis. Overshoots and oscillations as reported by the motor encoder tapes (Figure 2) would to some extent be absorbed by the elasticity of the model and plunge rods, but there is no direct data on the airfoil position itself.

PIV resolution was 88 pixels/cm. For 32x32 pixel windows with 16x16 overlap, this results in 84 velocity vectors per the 152mm chord length. Because of laser reflections from the model surface (polished stainless steel, painted flat-black) and lack of corrections for PIV windows which intersect with the model surface or for near-wall pixel regions saturated due to laser reflections, data closer than one window length to the airfoil wall – approximately 0.02c – are not reliable. This renders problematic a direct comparison of PIV and CFD in attached boundary layers, but offers sufficient detail to make comparisons regarding shear layers bounding laminar separation bubbles, and shed vortical structures.

The installation of the model and schematic of the rig assembly are shown in Figure 1. In the photograph, the model is inside the test section, but the glass walls are not visible. In the schematic, the pitch/plunge rig’s linear motors are atop a plate above the test section’s free surface, and the test section is not shown. With a 45cm wide by 61cm high test section, blockage based on projected frontal area of the model at maximum pitch amplitude is 6%. Gaps between the model tips and the tunnel walls were approximately 1.0mm on each end of the model. To obtain the “most 2-dimensional” flowfield, the PIV light sheet was placed at the 3/4 span location; that is, approximately halfway between the plunge rods and the tunnel wall. Light sheet thickness was approximately 2mm, though the large field of coverage (up to 45 cm) makes precise collimation of the light sheet difficult.

In addition to PIV, dye injection with food-coloring was used for qualitative visualization of flow separation. For dye injection a fiberglass model with c = 200mm was used. Re is the same as with the smaller model used for PIV, but blockage is of course higher. Dye was injected through ports buried in the model, ejecting out of the leading edge and near the trailing edge, at approximately the same spanwise position as the light sheet location for the PIV model. For the dye injection results, photographs were rotated to orient the airfoil’s suction side to towards the upper side of the image. A page of graph paper taped to the side of the test section away from the camera gives a rough orientation of the size of flowfield structures marked by the dye streaks.

![Figure 1. SD7003 airfoil used for PIV experiments, installed in HFWT test section, with plunge rods in position prior to a run (left); and schematic of pitch/plunge rig and airfoil model (right).](image)

In the experiments, the model was started from rest at φ=0°. After 5 periods of plunge, the motion was stopped. Camera framing rate at full resolution (max ~ 2.5 frames/second) would not support multiple PIV image
pairs per period of motion. Therefore, only one PIV pair was acquired per plunge period. By suitable selection of trigger phase, a PIV image pair was acquired at each of eight equally spaced phases per period of plunge oscillation. This was repeated 10 times per phase, for a total of 80 runs. After ensemble averaging across the 10 realizations, the data set was reassembled phase-by-phase to produce 40 velocity data records in sequence (8 phases over 5 periods). Vorticity was calculated by explicit differentiation of cubic spline fits to the velocity field for each velocity record prior to ensemble averaging. The PIV acquisition was limited to the single case of FR = 2, with pitch pivot at x/c = 0.5.

III. Results

We begin with the case of FR = 2, and pitch pivot point at x/c = 0.5. Computed lift and drag coefficient time history, superimposed on the angle of attack time history inferred from experiment, is shown in Figure 2. The rise in the total angle of attack curve (blue curve in Figure 2) at t/T~0 is followed by small oscillations, which damp out by t/T ~ 0.1 (corresponds to φ = 36°). For clarity, the value of thrust coefficient in Figure 2 was multiplied by a factor of 5. In the computation there are of course no such oscillations in angle of attack.

While there is a slight difference in the lift coefficient at t/T = 0 vs. t/T = 1, 2, and so forth, the difference is very small; it is even smaller in the respective time trace for drag. For whatever reason, the computation is not capturing the transient response in aerodynamic loads associated with motion startup, and the force coefficients appear to be periodic. Referring for convenience to the first plunge period, t/T = 0 through 1, we can apparently associate extrema in force coefficients to extrema in α_T, as follows: C_T has its main peak at t/T ~ 0.75, shortly after a negative peak in α_T; and another peak at t/T ~ 0.45, shortly after a positive peak in α_T. C_L has its main positive peak at t/T ~ 1, where α_T has its secondary peak; and the negative peak in C_L is shortly after the main positive peak in α_T. It appears, loosely speaking, that extrema in force coefficients lag very slightly extrema in α_T, though attempt to discern a causal relationship would be pure speculation at this point.

Turning towards the flowfield itself, vorticity contours based on PIV and computation, together with dye injection, are compared in Figure 3. Data from CFD immediately at motion onset is not shown. Instead, the upper righthand corner of Figure 3 shows a schematic of the airfoil pitch-plunge position history over one period of plunge (two periods of pitch). Vorticity contour values between -6 and +6, with the usual normalization by free stream velocity and airfoil chord, are blanked out for clarity, in both PIV and CFD. Data are shown for the following plunge phases of motion: φ = 0° (PIV and dye injection only), 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°, 405°, 450°, 495°, 540°, 585°, 630°, 675°, 720°, 1080°, and 1440°. φ = 360° completes one period of plunge oscillation after motion onset, 720° completes two periods, and so forth. Data from PIV is missing in the “shadow” of the laser light

Figure 2. Time trace of computed forces, superimposed on time trace of angle of attack. The angle of attack curves as shown are from the linear motors’ encoder tapes in the experiment: geometric pitch and total effective angle of attack.
sheet, on the pressure side of the airfoil. This is unfortunate, as vortex pairs shed from the leading edge occasionally convect to the airfoil’s pressure side, and thus out of view from the PIV. Images from dye injection become ambiguously clouded after $\phi = 720^\circ$, due in part to mixing of the dye and in part to three-dimensional effects of spanwise flow and tilting in shed vortices. Spanwise effects for the nominally 2D experiment tend to be very strong for large reduced frequencies, as reported in earlier work for pure-plunge experiments\textsuperscript{6}.

Comparison between CFD and experiment is quite good at early phases of motion, but degrades with elapsing time from motion onset. From $\phi = 45^\circ$ (the first CFD vorticity snapshot available) through $\phi = 225^\circ$, agreement between PIV and CFD is very close. Unlike in the force time history, the computational vorticity contours show a definite start-up process, and in fact never achieve what could be called relaxation to periodicity. By $\phi = 630^\circ$ divergence between experiment and computation becomes significant; the computation predicts a strong leading edge vortex pair just above the airfoil suction side, whereas in the PIV there is no such discernable concentration of vorticity, nor is there a clump of dye identifiable with such a vortex pair. At the trailing edge and in the near wake, PIV and CFD remain comparable through $\phi = 720^\circ$. We note that because the PIV results are ensemble-averaged, shed structures with poor repeatability will be attenuated in the average. The dye injection is, of course, a single snapshot.

Comparing $\phi = 720^\circ$, 1080$^\circ$ and 1440$^\circ$, the PIV shows strong repeatability. Dye streaks are notionally repeatable, but mixing and resulting dye diffusion renders interpretation of coherent dye concentrations impossible at $\phi = 1080^\circ$ or 1440$^\circ$. Therefore in the experiment the flow appears to relax to periodicity – if the ensemble average is to be believed – by two periods of plunge after motion onset. The CFD, on the other hand, shows continuing evolution of vortex shedding and increase in the number of discernable structures in going from 720$^\circ$ to 1080$^\circ$ and 1440$^\circ$.

The experiment in particular shows the curious phenomenon of shedding a vortex pair upstream from the leading edge, for example at $\phi = 450^\circ$. The wake behind the trailing edge meanwhile, is not parallel to the free stream but is biased towards the downwash direction. Such wake bias has been observed before for pure-plunge oscillations, for example by Jones et al\textsuperscript{8}.

<table>
<thead>
<tr>
<th>PIV</th>
<th>Dye Injection</th>
<th>Computation</th>
</tr>
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<tbody>
<tr>
<td><img src="image1.png" alt="PIV" /></td>
<td><img src="image2.png" alt="Dye Injection" /></td>
<td><img src="image3.png" alt="Computation" /></td>
</tr>
</tbody>
</table>

Note: Plunge amplitude exaggerated
In an effort to begin bridging the gap in Reynolds number between computation and experiment, the computation was repeated for Re = 150 and 600 (Figure 4). Not surprisingly, with increasing Re one ones more fine-scale structures. But as in the comparison between computation and experiment, the Re = 150 and Re = 600 computations show good agreement for small φ, but diverge appreciably by φ = 720°.

<table>
<thead>
<tr>
<th>φ = 1440</th>
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<tr>
<td><img src="image1.png" alt="Image" /></td>
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</table>

Figure 3. FR = 2, pitch pivot point at x/c = 0.5. Vorticity contours from PIV (left) and dye injection (middle), at Re = 10,000. Vorticity contours from CFD at Re = 300 (right)
Figure 4. Vorticity contours for CFD at Re = 150 (left) and Re = 600 (right).

The distinction between the computed vorticity contours for Re = 150, 300 and 600 is completely missing in the force time history. Figure 5 shows lift and drag coefficient for three periods of plunge after startup. As in Figure 1, the response is strongly periodic with no discernable startup effect, and there is little difference amongst the three families of curves. It appears, therefore, that while integrated aerodynamic force is periodic, the evolution of separated vortical structures in the flowfield is not. Evidently, said vortical structures contribute little to the integrated aerodynamic force.

Figure 5. Time history of lift and drag coefficient for Re = 150, 300 and 600, for three periods of plunge after motion onset.
Next, we consider the effect of the pitch pivot point location. Varying the pitch pivot point will strongly affect $\alpha_T$. Figure 6 compares dye injection at Re = 10000 with CFD at Re = 300 at two phases of motion in the first period after motion onset ($\varphi = 180$ and 360, again based on plunge frequency), for four pivot point locations: $x/c = 0$, 0.25, 0.75 and 1.0. Force time history from CFD is given in Figure 7.

![Images of dye injection for different pivot points](image-url)

$x/c = 0.0, \varphi = 180$

$x/c = 0.0, \varphi = 360$

$x/c = 0.25, \varphi = 180$
$x/c = 0.25, \phi = 360$

$x/c = 0.75, \phi = 180$

$x/c = 0.75, \phi = 360$

$x/c = 1.0, \phi = 180$
Figure 6. Sweep of pitch pivot point: x/c = 0, 0.25, 0.75 and 1.0; dye injection at Re = 10,000 (left) and CFD vorticity contours at Re = 300 (right). The x/c = 0.50 case is shown in Figure 3.

Figure 7. Time history of lift and drag coefficients for sweep of pitch pivot point: x/c = 0, 0.25, 0.5, 0.75 and 1.0. The timebase is from motion onset through the end of three plunge periods.

Returning to the usual case, FR = 1, dye injection at Re = 10000 is compared with CFD at Re = 300 in Figure 8, and the computed force time history is given in Figure 9. Now we recover the familiar reverse Karman vortex street in both dye injection and CFD vorticity contours. Again, the dye injection shows that the flowfield relaxes to periodic by φ = 720°. This time the computation is essentially periodic by φ = 720° as well, with little difference between φ = 720° and φ = 1440°. The main difference between dye and CFD vorticity is the former’s larger wake width.

The computed force time history is again stubbornly periodic, with no startup transient. But in marked distinction to FR = 2, for FR = 1 the force coefficients are nearly sinusoidal. Peaks in Cₜ closely track the αₜ time history in phase and sign. Cₗ has twice the frequency of the αₜ curve.
Figure 8. Frequency ratio FR = 1, pivot point at x/c = 0.5. Dye injection at Re = 10,000 (left) and CFD vorticity contours at Re = 300 (right) for four periods of plunge.

Figure 9. Force coefficient time history vs. geometric and total effective angle of attack: pitch-plunge frequency ratios of 1 and 2, pivot point at x/c = 0.5.
IV. Summary

2D airfoil pitch-plunge where the pitch frequency is twice the plunge frequency produces strong excursions between angle of attack peaks, and consequently strong and unusual behavior in vortex shedding and aerodynamic force time history. Agreement in observable flowfield structures between particle image velocimetry at Re = 10000 in a water tunnel, vs. immersed boundary computation at Re = 300, is quite good for two plunge periods after motion startup, but degrades thereafter. In general it seems to be the case that for the high-frequency massively-unsteady problems under consideration, the effect of Reynolds number is benign for short times after motion startup. Curiously, computed force coefficients showed strong periodicity, while vorticity contours did not. Close agreement in phase between force time history and total angle of attack time history, while consistent, is perhaps fortuitous. Upon comparison with the usual case where pitch and plunge frequency are equal, the reverse Karman vortex street is recovered.

V. Acknowledgement

This work is supported under AFRL/DASGI Ohio Student-Faculty Fellowship program.

VI. References


3 http://www.ae.uiuc.edu/m-selig/ads/coor d_database.html