Vortex Formation and Aerodynamic Force of Low Aspect-Ratio Plate in Translation and Rotation

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A low aspect-ratio plate under translational or rotational motion in a wide range of angles of attack is studied using Direct Numerical Simulation (DNS). Vortex formation and aerodynamic performance generated in the two motions are discussed. The aerodynamic forces obtained from two methods are calculated and compared. One method is the typical surface integration of pressure and shear stress tensor; another method is the time derivative of the volume integration of vorticity moment. The calculated forces from the two methods match reasonably well.

Nomenclature

$C_x$ = Lift coefficient in global x-direction
$C_y$ = Drag coefficient in global y-direction
$r$ = Position vector
$S$ = Wing projected area
$Re$ = Reynolds number
$U$ = Characteristic velocity
$\alpha$ = Angle of attack
$\omega$ = Vorticity

Introduction

The aerodynamic or hydrodynamic performance of three-dimensional wings or flapper has been extensively investigated both experimentally (Dickinson [1], Birch et al. [2], Dickinson [3], Fry et al. [4], Birch et al. [5], Sane [6], etc.) and numerically (Dong et al. [7], Taira et al. [8], etc.). However, some of the fundamental issues of low aspect ratio plates under either translation or rotation are still not fully understood yet. With direct numerical simulation (DNS), the present work will focus on the comparison of vortex development and aerodynamic performance of rotating and translating low-aspect ratio plates.

The aerodynamic force can be calculated from the object surface integration of pressure and shear stress tensor, which may be directly solved from the Navier-Stokes equations using various numerical approaches. Some researchers may also solve for the vorticity first through the vorticity equation, then calculate the pressure and shear stress from the known vorticity field, especially for two-dimensional flow. Wu [10] developed a theory to compute the aerodynamic force directly from the vorticity field.

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However, Wu’s method has been relatively rare used in the fluid dynamic problems with moving boundary involved [11, 12]. In this paper, we numerically study the translation and rotation of a low aspect ratio plate at various angles of attack. Wu’s theory is then applied to the calculation of aerodynamic forces, which are also compared with results from the surface integration. In the following, we first introduce the DNS solver used in the current study, and then briefly summarize Wu’s theory, which followed by a simulation of a two-dimensional hovering plate. The three-dimensional translating and rotating plates are then studied respectively.

**Governing Equation and Numerical Method**

The incompressible Navier-Stokes equations, written in tensor form

\[
\frac{\partial u_i}{\partial x_i} = 0, \quad \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i^2},
\]

is solved using finite-difference based Cartesian grid method (Dong et al.[7]). The method employs a second-order central difference scheme in space and a second-order accurate fractional-step method for time advancement. The Eulerian form of the Navier-Stokes equation is discretized on a Cartesian mesh and boundary conditions on the immersed boundary are imposed through a “ghost-cell” procedure. Validations of codes can be found in Mittal et al. [9].

**Impulse Based Force Calculation**

Wu [10] derived the expressions for the aerodynamic force and moment on a body moving in an incompressible viscous flow. The non-dimensional force exerted on the body by fluid \( F \), can be written as:

\[
F = -\frac{d\gamma_f}{dt} + \frac{2}{\rho c} \frac{d}{dt} \int_V \nu dV,
\]

where \( \gamma \) is the first moment of vorticity and is defined as:

\[
\gamma = \int_V r \times \omega \, dV,
\]

in which \( r \) is the position vector and \( \omega \) is vorticity vector. The above integration can be performed in both fluid domain and solid body. The first term in Eq. (2) is the time rate of change of first moment of the vorticity field; the second term is the inertia force due to the mass of fluid displaced by the solid body. From Eq. (2), it can be seen that the force generation is related to both the vorticity magnitude and its distribution [12]. Also, for a very thin plate, the contribution from the second term is negligible.

**Results**

**A. Two-dimensional plate in hovering**

In this section, we study the hovering motion of a two-dimensional plate. The aerodynamic forces calculated from surface integration of pressure and shear stress tensor are compared with those from impulse based formulation. The kinematics of a hovering plate is prescribed by:

\[
x(t) = \frac{\Delta s}{2} \cos(2\pi f t), \quad y(t) = 0,
\]
If the parameter $C_\theta$ is 0, then the inclination angle is simply reduced to sinusoidal function:

$$\theta(t) = A_\theta \sin(2\pi ft + \phi_\theta)$$

The kinematics of the hovering plate with $C_\theta = 0$ can be seen in Figure 1, in which the peak to peak amplitude $A_x$ is 4. The inclination angle amplitude $A_\theta$ is 45 deg. and phase angle $\phi_\theta$ is 0 deg. The blue and red colors represent the plate downstroke and upstroke respectively. The variation of the inclination angle is also shown with $C_\theta$ as a parameter. It can be seen that the rotational speed at the stroke reversal gets faster as the parameter $C_\theta$ increases. With $C_\theta$ being set to 4, the time history of the inclination angle almost shows a square-wave. The rising edge of the trajectory would be even sharper with higher $C_\theta$. The Reynolds number $Re$ used in the current study is 200.

In Figure 2, the history of aerodynamic force coefficient is shown for three fully flapping cycles. The characteristic speed $U$ used for the force coefficients is $\pi A_x f$. The forces calculated from surface integration of pressure and shear stress tensor, and from time derivative of vorticity moment are compared. It can be seen that the force calculations from the two methods match well.

**Figure 1 Kinematics of a hovering plate**

**Figure 2** Force comparison using surface integration and impulse based method. The plate is undertaking a hovering motion, with the peak to peak amplitude $A_x = 4$, inclination angle amplitude $A_\theta = 45$ deg, and $\phi_\theta = 0$. The Reynolds number is 200.

**B. Low-Aspect Ratio Plate in Translation**

In this section, we numerically study the translation of a rectangular plate with low aspect-ratio $AR$, which is defined as the ratio of the span (b) to the chord (c). The plate moves with a unified speed (U)
from the right to the left, as shown in Figure 3. The aspect-ratio AR is 2 and the plate has a 5% thickness-to-chord ratio. The Reynolds number Re based on chord length and linear translation speed is fixed at 200. Translation distance in the current study is set to 3. The angle of attack ($\alpha$) is the angle between the plate inclination and the horizontal direction.

The force coefficients are defined as $C_X = 2F_X/\rho U^2S$ and $C_Y = 2F_Y/\rho U^2S$ respectively in horizontal and vertical direction. $F$ is the aerodynamic force calculated from either the surface integration or the vortex moment method; $S$ is the wing surface area and equals to 2 in the current case for a plate with aspect-ratio 2. Figure 4 has presented the history of force coefficients of a translating plate moving with constant speed ($U = 1$) at various angle of attack. The Reynolds number Re is 200. The plate experiences high impulse at the beginning of motion. It can be seen that the force coefficient $C_X$ monotonically increases with the angle of attack. For relatively low angle of attack (15 deg. and 30 deg.), $C_X$ approaches steady value quickly. The maximum vertical force coefficient $C_Y$ is obtained at $\alpha = 45$ deg. Note the plate at low angle of attack (15 deg.) and the high angle of attack (75 deg.) generates comparable lift during the translation.

Figure 4. The force coefficients of a translating plate (AR=2) at various angle of attack ($\alpha$). Re =200

Figure 5 shows 3-D perspective view and side view of the wake structures using the isosurface of Q-criteria at the mid-stroke and at the end of the stroke. In linear translation case, the ring is primarily formed by symmetric tip vortices and trailing edge vortex (TEV), leading edge vortex (LEV) will shed and form another ring when angle of attack increases. At t= 1.5, the LEV at the mid-span covers about one half of the plate in the chord-wise direction. At $t = 3.0$, the LEV is fully developed. The vortex ring composed by the TEV and the tip vortices forms a 30 deg. angle with respect to the horizontal x-z plane.
Figure 5. Wake structure of linearly translating plate at (a) t=1.5, (b) t=3.0, $\alpha = 45$ deg.

Figure 6. Vorticity on cross sections at various span-wise locations of a translating plate (t = 3)

Figure 6 has shown the vorticity contour in the cross section at three various span-wise locations for a translating plate, where $z/b = 0.5$ is the mid-span, and $z/b = 0.875$ is close to the tip of the plate. The snapshot is taken at $t = 3$. Apparently the leading edge vortex is the strongest at $z/b = 0.5$ among the three locations. Figure 7 shows the force coefficients calculated from object surface integration and from the time derivative of volume integration of the vorticity moment in the whole flow domain. We can see that both $C_X$ and $C_Y$ fit reasonable well during the translation process in the current study.

Figure 8 presents the force generated on cross sections at various spanwise locations. It can be seen from Figure 8 (a) that the mid-span ($z/b = 0.5$)
experiences highest drag compared to other span locations over the whole translation process. At \( t=3 \), the mid-span generate maximum lift and the \( z/b = 0.875 \) cross section generates minimum lift among the three span-wise locations have been investigated. This observation is consistent with the lifting line theory for a three-dimensional wing with finite span in steady state.

![Graph](image)

(a) Horizontal direction  
(b) Vertical direction

**Figure 8. Force generated on cross sections at various spanwise locations**

C. Low-Aspect Ratio Plate in Rotation

The rotation of a rectangular plate with low aspect-ratio AR=2 is studied in this section. The plate rotates with a constant angular velocity from the right to the left, as shown in Figure 9. The linear velocity at the mid-span is 1. The thickness-to-chord ratio is 5%. The Reynolds number (Re) based on the chord length and linear speed at the mid-span is 200. The total rotation angle in the current study is \( \pi \), and the rotation axis is fixed at the wing root. The angle of attack \( \alpha \) is the angle between the plate inclination and the horizontal (XZ) plane. At the start of the rotation, the span-wise direction of the plate is in parallel with the x-axis of the global coordinate system, and points to the positive x-direction from the root to the tip before the rotation. After rotation, the plate points to the direction of negative x-axis.

Figure 10 shows the force coefficients of the plate rotating at various angles of attack. The convex shape of \( C_x \) history is consistent with the rotating plate kinematics, where the plate is in parallel with x-direction at the beginning; and the plate is perpendicular to the x-direction in the middle of rotation. The coefficient \( C_x \) increases with angle of attack monotonically.

![Diagram](image)

**Figure 9. A plate of AR=2 in rotation, from the right to the left with constant angular velocity.**

It is interesting to note that the rotating plate (Figure 10b) has much higher lift production compared with linearly translating plates. We can see that for small and medium angle of attack (\( \alpha \leq 45 \) deg. in the
current study of Re = 200), the lift coefficient can keep at a constant value once the initial starting process fades out. Recalling the decrease of \( C_L \) in the translation case (Figure 4b), we can deduce that the spanwise flow in the rotation case helps the stabilization of the leading edge vortex, and therefore the lift generation. At high angle of attack (e.g. \( \alpha = 60, 75 \) deg.), \( C_L \) decreases after an initial increasing in the beginning of rotation, a similar phenomenon has been seen in the case of translating plate. Therefore, the stabilization of LEV due to the spanwise flow is weakened when the angle of attack is too high. In addition, the comparison of \( C_L \) for \( \alpha = 60 \) deg. and \( \alpha = 75 \) deg. in Figure 10b shows that the higher the angle of attack is, the faster lift coefficient drops.

![Figure 10. The force coefficients of a rotating plate (AR=2) at various angles of attack (\( \alpha \)). Re =200](image)

![Figure 11. Wake structure of a rotating plate at (a) t=1.5, (b) t=3.0, angle of attack 45 deg.](image)

Figure 11 presents the vortex structure generated by a rotating plate with 45 deg. angle of attack. The vortex ring formed in the uniform rotation case consists of leading edge vortex, tip vortex and trailing edge vortex. The leading edge vorticity shows a strong spanwise convection from the root of the plate to the tip of the plate. As discussed in Ellington et al. [13], the spanwise flow in uniformly rotating plates stabilize the leading-edge vortex and make it attach to the plate. From the side view of Figure 11b, the vortex ring composed by the tip vortices and TEV forms an angle around 10 deg. with respect to the
horizontal plate. In contrast to the vortex ring in the translational plane, the vortex ring in the rotating plate can induce stronger downwash flow and therefore makes higher lift generation.

Figure 12. Force calculated from surface integration and impulse method ($\alpha = 45$ deg.)

For a plate rotates with 45 deg. angle of attack, Figure 12 shows the force calculated from pressure and shear stress surface integration and the time derivative of volume integration of vorticity moment. The comparison is reasonably well.

Figure 13 presents the vorticity contour in spanwise cross sections, where $z/b = 0.875$ is close to the plate tip and $z/b = 0.25$ is close to the root. It can be seen that the distance from the trailing edge vortex to the plate in the section near the tip of the plate is far closer than that in the section near the root. The TEV contribution to the lift generation is under future investigation.

Figure 14 shows the comparison of plate surface pressure between linear translation and uniform rotation at $t = 1.5$ and $t = 3.0$ respectively, both plates under 45 deg. angle of attack. From back view of the plate, rotation motion generates significant low pressure area at the leading edge near the tip due to the attached LEV. Along with the time increment, this low pressure area was maintained, which may be one reason that explains the maintenance of lift in the rotation plate. For the translating plate, the area of

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low surface pressure region in the suction side decreases as time increases from 1.5 to 3.0, as shown in the back view of the plate. Thus, the lift coefficient drops off (Figure 4) when time is greater than 1.5 in the current study.

![Figure 14. Comparison of plate surface pressure between linearly translating plate (left two) and uniformly rotating plates (right two) at (a) mid-stroke, (b) the end of the stroke. The angle of attack is 45 deg.](image)

**Conclusion**

In this paper, translational and rotational motions of a plate with aspect ratio 2 are numerically studied in a wide range (from 15 deg. to 75 deg.) of angle of attack. The Reynolds number used in this study is 200. The vortex formation, aerodynamic performance, and flow analysis are discussed. For both translation and rotation, the plate at 45 deg. generates highest lift in each motion category. For the plate under translation, lift can hold at a constant level only when the angle of attack is small enough. At higher angle of attack, e.g. $\alpha$ is 30 deg. or above, the lift generation starts to drop off after initial starting process. In addition, the higher the angle of attack is, the earlier this lift degradation starts. For the plate under rotation, the spanwise flow can stabilize the leading edge vortex; lift generation thus keeps at a constant level even when the angle of attack is as high as 45 deg. The lift degradation occurs when $\alpha$ is 60 deg. and 75 deg. Also, the magnitude of lift reduction is greater for higher angle attack. The time derivative of vorticity moment in the flow field is calculated and aerodynamic forces are obtained from impulse method. It has been shown that the forces estimated from vorticity moment match well with those from object surface integration of pressure and stress tensor. Detailed study on force contribution of respective vortex distribution or structure is under further investigation.
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References


